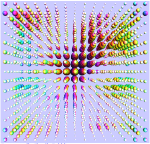
# Lattice gauge theory at the electroweak scale



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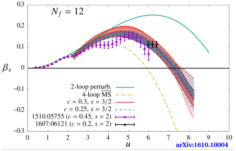


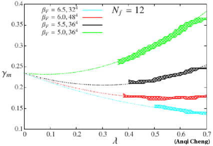


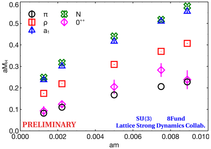



David Schaich (U. Bern)


Strong dynamics at the electroweak scale
Montpellier, 6 December 2017


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# Overview and plan

Lattice gauge theory is a broadly applicable tool
to study strongly coupled systems

Especially important
when QCD-based intuition may be unreliable

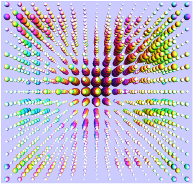
A high-level summary of lattice gauge theory

$\beta$ functions and anomalous dimensions

Light scalar from near-conformal dynamics

More possible topics for discussion

- Electroweak $S$ parameter
- Composite dark matter
- Multi-rep. composite Higgs UV completions
- …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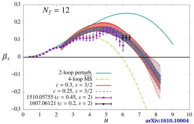
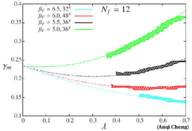
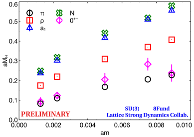
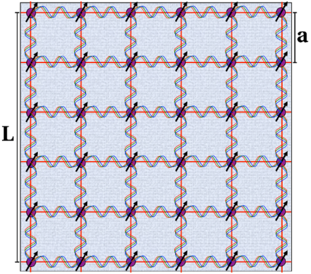


# The essence of lattice gauge theory

Lattice discretization is a non-perturbative regularization of QFT



P. Vranas LLNL

Formulate theory on finite, discrete euclidean space-time $\longrightarrow$ **the lattice**

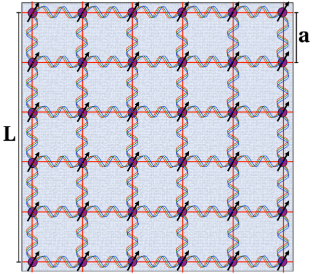
Spacing between lattice sites ("$a$") $\longrightarrow$ UV cutoff scale $1/a$

Removing cutoff: $a \to 0$ (with $L/a \to \infty$)

Finite number of degrees of freedom $\left(\sim 10^9\right)$
$\longrightarrow$ numerically compute observables via importance sampling

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}U \;\; \mathcal{O}(U) \;\; e^{-S[U]} \quad \longrightarrow \quad \frac{1}{N} \sum_{k=1}^{N} \mathcal{O}(U_k)$$

# Features of lattice gauge theory

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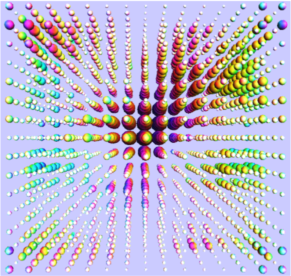


(Image credit: Claudio Rebbi)

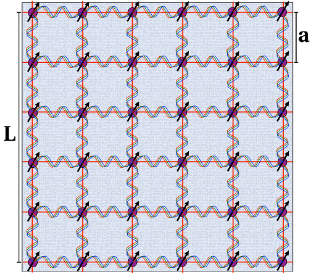
Fully non-perturbative predictions from first principles (lagrangian)

Fully gauge invariant—no gauge fixing required

Applies directly in four dimensions

Euclidean SO(4) rotations & translations ($\longrightarrow$ Poincaré symmetry) recovered automatically in the $a \to 0$ continuum limit

# Limitations of lattice gauge theory



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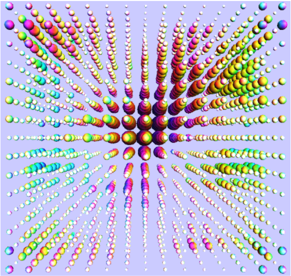
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Need UV completion, (usually) include only strong sector

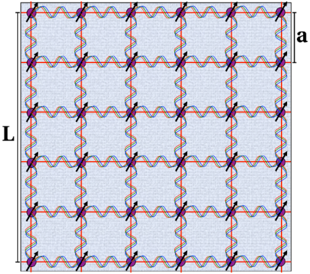
Finite volume (usually) needs to contain all correlation lengths

$\longrightarrow$ unphysically large masses extrapolated to chiral limit via EFT

Chiral symmetry of lattice fermion operator complicated

Obstructions to chiral gauge theories, real-time dynamics, susy

# Limitations of lattice gauge theory

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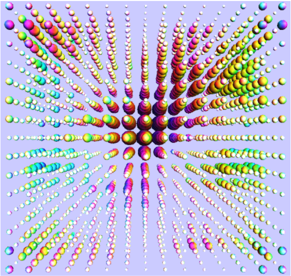

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# Lattice fermion discretizations

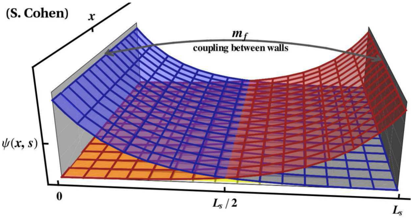
Tension between chiral symmetry vs. ‘doubling’ of lattice fermions

**Naive** $\longrightarrow$ $16F$ continuum fermions from $F$ lattice fields,
large $\mathrm{U}(4F)_V \times \mathrm{U}(4F)_A$ chiral symmetry

**Staggered** $\longrightarrow$ $4F$ continuum fermions, $\mathrm{U}(F)_V \times \mathrm{U}(F)_A$ chiral symm.

**Wilson** $\longrightarrow$ $F$ continuum fermions, no chiral symmetry

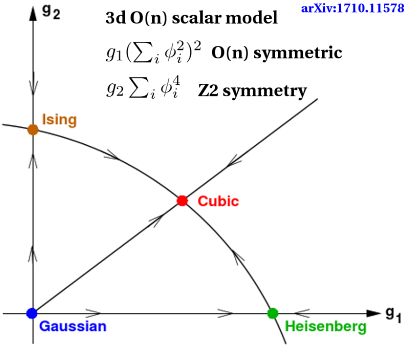
**Domain wall** $\longrightarrow$ $F$ continuum fermions,
lattice “remnant” $\mathrm{SU}(F)_V \times \mathrm{SU}(F)_A$ chiral symmetry



# Symmetries of lattice fermions

Different lattice symmetries for fixed $N_F$ continuum fermions

| **Domain wall** | **Staggered** | **Wilson** |
|---|---|---|
| $SU(N_F)_V \times SU(N_F)_A$ | $U(N_F/4)_V \times U(N_F/4)_A$ | None |




All $\longrightarrow$ same UV continuum limit
('lattice universality')

Possibility
different lattice symmetries
$\longrightarrow$ different IR dynamics?

Example of 3d $O(n)$ scalar model

# Lattice gauge theory beyond QCD

Lattice calculations especially important for non-QCD strong dynamics

Exploratory investigations of representative systems
$\longrightarrow$ elucidate generic dynamical phenomena, connect with EFT

arXiv:1309.1206 arXiv:1510.05018 arXiv:1701.07782


## Executive Summary

- Use the Higgs boson as a new tool for discovery
- Pursue the physics associated with neutrino mass
- Identify the new physics of dark matter
- Understand cosmic acceleration: dark energy and inflation
- Explore the unknown: new particles, interactions, and physical principles.

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# Non-QCD strong dynamics

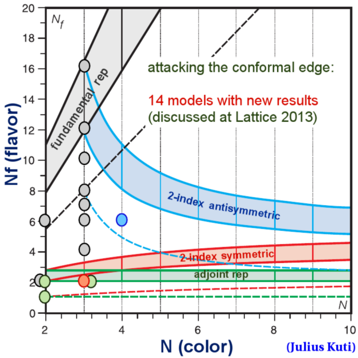
## Two main directions (not mutually exclusive)

Near-conformal dynamics from many fermionic d.o.f.

$\longrightarrow$ large number of fundamental fermions or a few in a larger rep

Different symmetries from different gauge group or reps

$\longrightarrow$ (pseudo)real reps for cosets SU($n$)/Sp($n$) or SU($n$)/SO($n$)



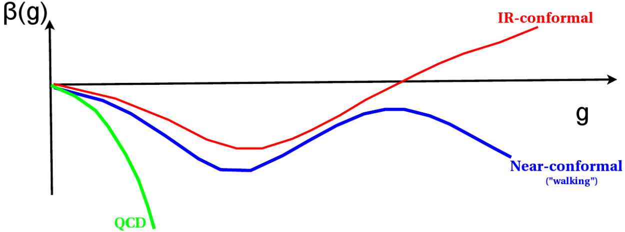

Today focus on near-conformality

Study a few representative systems,
look for similarities/difference vs. QCD

Start with non-perturbative $\beta$ function

# $\beta$ function motivation

$$\beta = \frac{dg^2}{d\log\mu^2} \longrightarrow \text{scale dependence of running coupling}$$




Perturbative $\quad \beta(g^2) = -\frac{g^4(\mu^2)}{16\pi^2}\left[b_1 + b_2\frac{g^2(\mu^2)}{16\pi^2}\right] + \mathcal{O}\left(g^8\right)$

Asymptotic freedom in UV $\longrightarrow$ $b_1 = \frac{1}{3}\left[11C_2(G) - 4N_F T(R)\right] > 0$

$b_2 < 0$ might give non-trivial conformal fixed point in IR

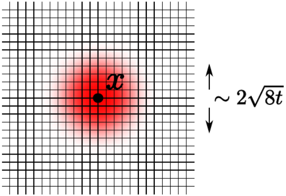
Banks & Zaks make argument rigorous for $b_1 \approx 0$

# Lattice $g^2$ for non-perturbative $\beta$ function

First step: Define measurable $g^2$ with scale given by lattice size $L$

Use Yang–Mills gradient flow
(integrating infinitesimal smoothing operation)

Local observables measured after "flow time" $t$
depend on original fields within $r \simeq \sqrt{8t}$



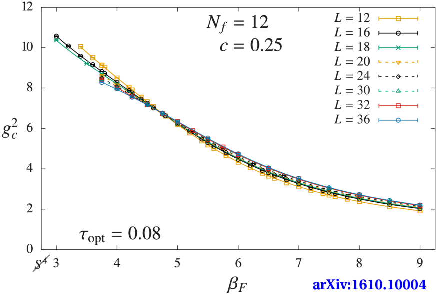

Flowed energy density $E(t) = -\frac{1}{2}\mathrm{Tr}\left[G_{\mu\nu}(t)G^{\mu\nu}(t)\right]$
perturbatively gives $g^2_{\overline{\mathrm{MS}}}(\mu) \propto t^2 E(t)$ with $\mu = 1/\sqrt{8t}$

Tie to lattice size by defining $g_c^2(L; a)$ at fixed $c = L/\sqrt{8t}$
(scheme dependent as expected)

# Step scaling for non-perturbative $\beta$ function

Next step: Scale change $L \longrightarrow sL$ gives discrete $\beta$ function

$$\beta_s(g_c^2; L) = \frac{g_c^2(sL; a) - g_c^2(L; a)}{\log(s^2)} \quad \xrightarrow{s \to 1} \quad -\beta\left(g^2(\mu^2)\right)$$




$N_F = 12$ staggered fermions,
bare coupling $\beta_F \simeq 12/g_0^2$

With $s = 3/2$ have

| | |
|---|---|
| $L = 12 \to 18$ | $16 \to 24$ |
| $20 \to 30$ | $24 \to 36$ |

$s = 2$ and $4/3$ also accessible

$g_c^2$ for all $L$ cross around $g_c^2 \approx 7 \longrightarrow \beta_s(g_c^2; L) = 0$

Does $\beta_s$ remain zero as $L \to \infty$?

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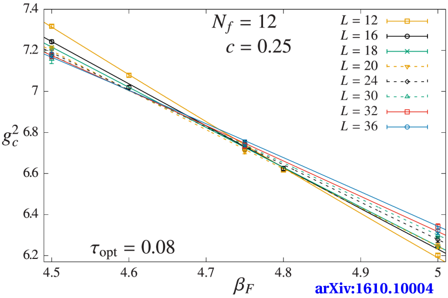


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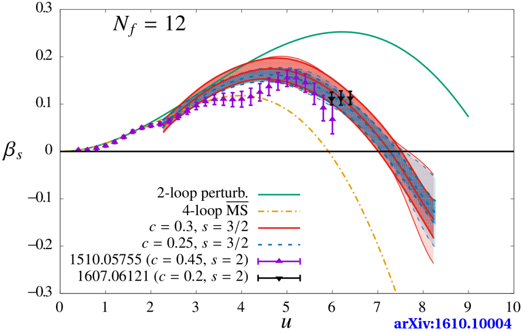
# Continuum extrapolation

Final step: Extrapolate $(a/L) \to 0$ to obtain continuum $\beta_s(g_c^2)$




$N_F = 12$ staggered results seemed broadly consistent

Even for different schemes and scale changes $s$

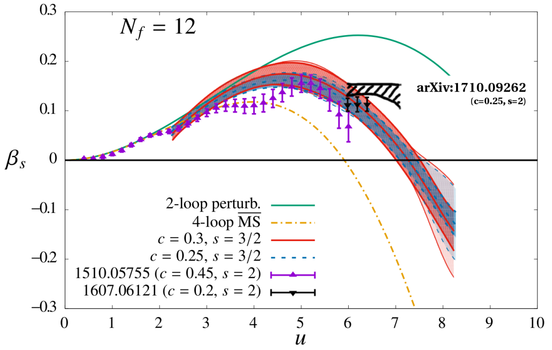
Slope at fixed point $g_\star^2 \approx 7.3$
$\longrightarrow \gamma_g^\star = -0.26(2)$
(scheme independent)

Simple $(a/L)^2 \to 0$ extrapolations fine near gaussian UV fixed point

May need $g_c^2(L; a) - g_\star^2 \propto L^{\gamma_g^\star}$ finite-size scaling near IR fixed point…

# Current status of staggered $N_F = 12$ $\beta$ function

Developing tension between two independent staggered analyses
$\longrightarrow$ not yet consensus about $N_F = 12$ fixed point




Same lattice symmetries
$\longrightarrow$ same fixed point

Despite details of
lattice action, analyses

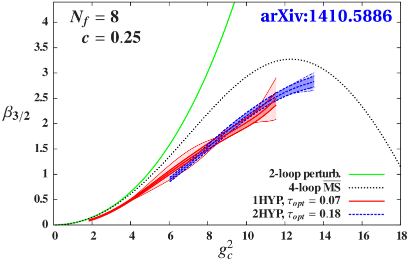
Main difference is
larger $sL \leq 56$ vs. 36

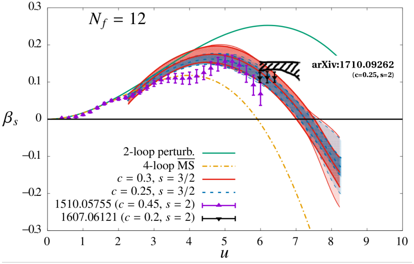
Tension related to $(a/L)^2 \to 0$ extrapolations vs. finite-size scaling?

# $\beta$ function wrap-up: Challenge I

$\beta$ function becomes very small as $N_F$ increases

Order of magnitude decrease for $N_F = 8$ (left) vs. $N_F = 12$ (right)

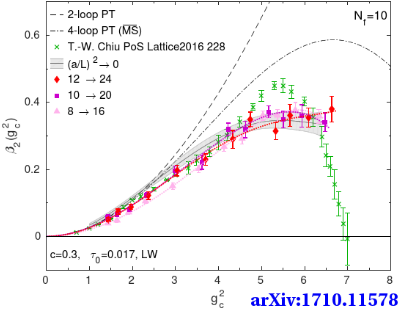


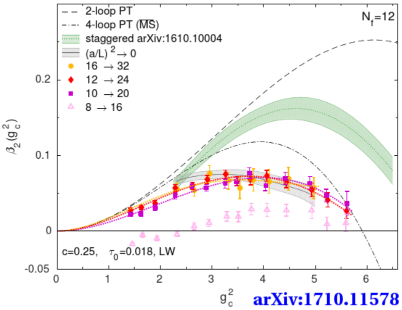




Hard to distinguish slow running vs. no running on finite lattices

# $\beta$ function wrap-up: Challenge II

Different symmetries of lattice fermions
$\longrightarrow$ IR fixed points in different universality classes?




Recently reported tensions between staggered vs. domain wall results
$\longrightarrow$ currently developing story

# Anomalous dimension motivation

At IR fixed point, universal anomalous dimensions $\gamma^\star$
$\longrightarrow$ scheme-independent critical exponents characterizing CFT

Large $\gamma$ wanted for fermion mass generation by new strong dynamics
(hopefully discussed in previous talk)

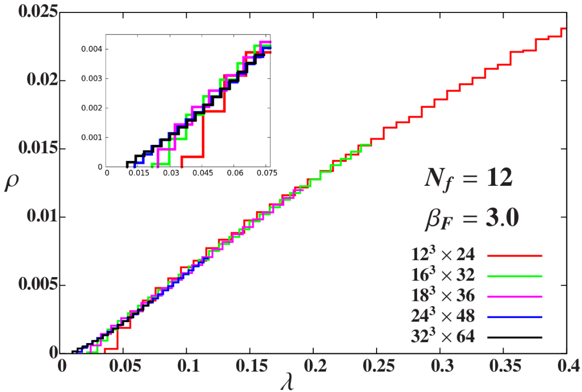
Near-conformality $\longrightarrow$ scheme and scale dependence negligible?

Plan: Focus on staggered $N_F = 12$ IRFP

- Already saw $\gamma_g^\star \approx -0.26$ from slope of $\beta$ function
- Extract mass anomalous dimension $\gamma_m^\star$ from Dirac eigenmodes
- Extract $\gamma_m^\star$ and $\gamma_g^\star$ from spectrum finite-size scaling
- Prospects for baryon anomalous dim. for partial compositeness

# $\gamma_m^\star$ from Dirac eigenvalue mode number $\nu(\lambda)$

$\mathcal{L} \supset \overline{\psi}\,(\not{D} + m)\,\psi \quad \longrightarrow \quad \not{D}$ eigenvalues sensitive to $\gamma_m^\star = 3 - \mathrm{d}[\overline{\psi}\psi]$




Histogram of eigenvalues
$\longrightarrow$ spectral density $\rho(\lambda)$

Integral is **mode number**

$$\nu(\lambda) = 2V \int_0^\lambda \rho(\omega) d\omega$$

Conformal FP: $\rho(\lambda) \propto \lambda^\alpha$
$\longrightarrow \nu(\lambda) \propto \lambda^{1+\alpha}$

Mode number RG invariant $\longrightarrow 1 + \gamma_m^\star = \dfrac{4}{1+\alpha}$ (Del Debbio & Zwicky)

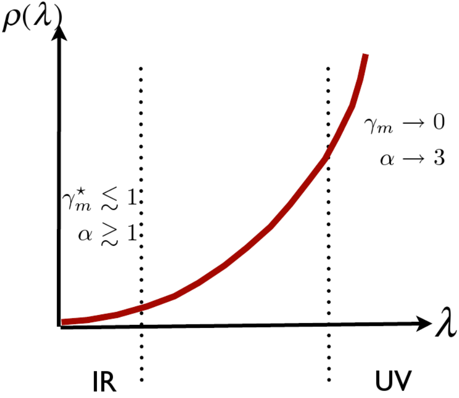
# Scale-dependent $\gamma_{\text{eff}}(\lambda)$ from eigenmodes

$\lambda$ defines energy scale $\longrightarrow$ $\nu(\lambda)$ gives effective $\gamma_{\text{eff}}(\lambda)$ at that scale

**UV:** Asymp. freedom $\Rightarrow \gamma_{\text{eff}}(\lambda) \to 0$
or $\alpha(\lambda) \to 3$

**IRFP** $\Longrightarrow \gamma_{\text{eff}}(\lambda) \xrightarrow{\lambda \to 0} \gamma_m^\star$

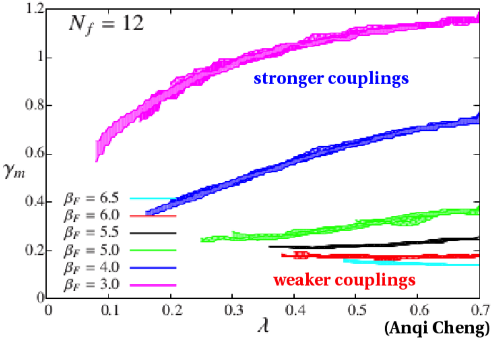
$\langle \overline{\psi}\psi \rangle \propto \rho(0) \neq 0 \Longrightarrow \alpha \to 0$,
breakdown of $\rho(\lambda) \propto \lambda^\alpha$




Monitor $\gamma_{\text{eff}}(\lambda)$ evolution from perturbative UV to strongly coupled IR

# $\gamma_{\text{eff}}(\lambda)$ from eigenmodes for $N_F = 12$

Fit $\nu(\lambda) \propto \lambda^{1+\alpha}$ in small range of $\lambda$ $\longrightarrow$ $1 + \gamma_{\text{eff}}(\lambda) = \dfrac{4}{1+\alpha(\lambda)}$




$\nu(\lambda)$ computed stochastically

Include fit ranges in error bands

Multiple $L^4$ volumes overlaid,
$L$-sensitive data dropped

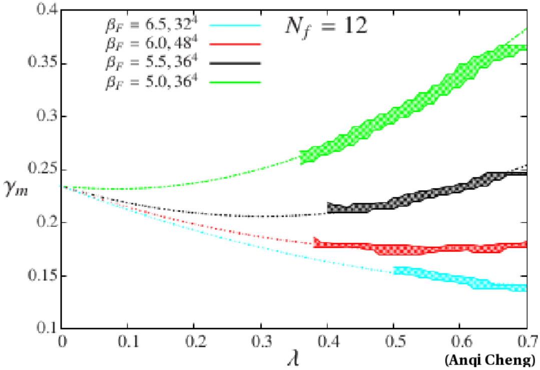
All systems have $\rho(0) = 0$

Strong dependence on irrelevant bare coupling $\beta_F \simeq 12/g_0^2$

$\gamma_{\text{eff}}$ increasing with $\lambda$ $\sim$ "backward flow" at strong coupling

# $\gamma_m^\star(\lambda)$ from eigenmodes for $N_F = 12$

Extrapolate $\lim_{\lambda\to 0} \gamma_{\text{eff}}(\lambda) = \gamma_m^\star$ at conformal IR fixed point




Zoom in on largest volumes, couplings closest to $g_\star^2$ (in this scheme)

Joint quadratic extrapolation $\longrightarrow \gamma_m^\star = 0.24(3)$

Uncertainty dominated by $\lambda \to 0$ extrapolation

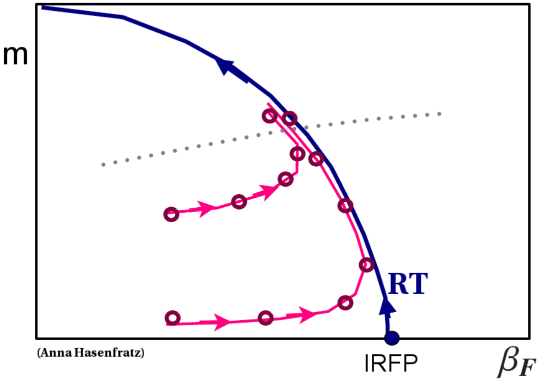
Single fit for some range of $\lambda > 0$ would give precise result but generally **not** $\gamma_m^\star$ at the $\lambda \to 0$ IR fixed point

# Wilson RG picture of finite-size scaling

Fermion mass $m$ is relevant coupling; gauge coupling $\beta_F$ is irrelevant

Increase $m$ and decrease RG flow ($L$)

$\longrightarrow$ same point on renormalized trajectory (RT)




Universal flow along RT

Correlation lengths depend on scaling variable

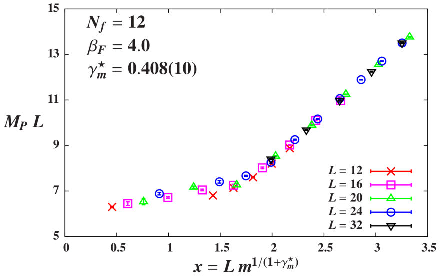
$$x \equiv L\, m^{1/(1+\gamma_m^\star)}$$

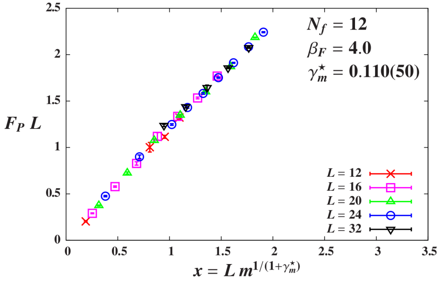
Assuming RG flow quickly reaches RT

# Naive finite-size scaling for $N_F = 12$

Correlation lengths depend on scaling variable $x \equiv L\, m^{1/(1+\gamma_m^\star)}$

$\longrightarrow \gamma_m^\star$ from optimizing **curve collapse** of $M_H L = f_H(x)$



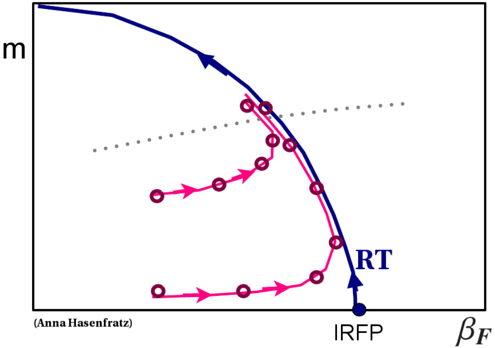

Curve collapse $\longrightarrow$ non-universal $\gamma_m^\star$ from different observables

Conformality requires universal $\gamma^\star$

$\longrightarrow$ corrections to scaling from near-marginal gauge coupling?

# Corrections to finite-size scaling

Slowly running gauge coupling $\longrightarrow$ RG flow may not reach RT
$\longrightarrow$ non-universal results from curve collapse




Leading correction to scaling:

$$M_H L = f_H(x, gm^{\omega})$$

where $\omega = -\gamma_g^{\star}/(1 + \gamma_m^{\star})$

Two-loop $\overline{\mathrm{MS}}$: small $\omega \approx 0.2$

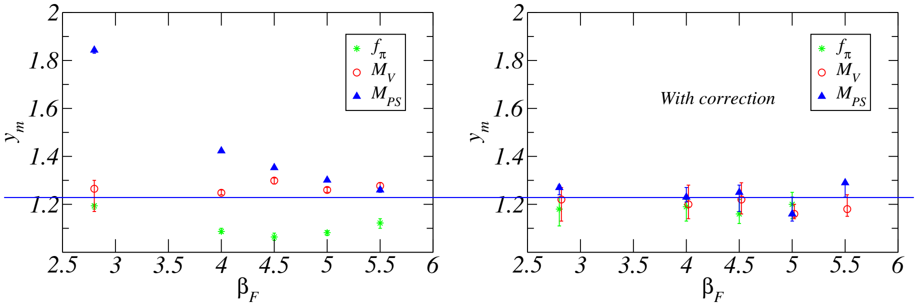
Hard to extract both $\gamma_m^{\star}$ and $\gamma_g^{\star}$ from curve collapse analyses
$\longrightarrow$ simplify $f_H(x, gm^{\omega}) \approx f_H(x)\left[1 + c_g m^{\omega}\right]$

# Consistent corrected finite-size scaling for $N_F = 12$

Approximate $M_H L \approx f_H(x)\left[1 + c_g m^{\omega}\right]$

$\longrightarrow$ consistent $\gamma_m^\star$ from all observables and $\beta_F$

Quality of curve collapse also improves



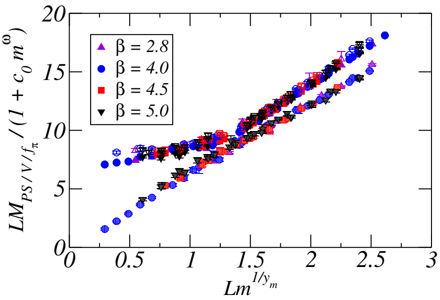

Can attempt combined analyses of multiple data sets. . .

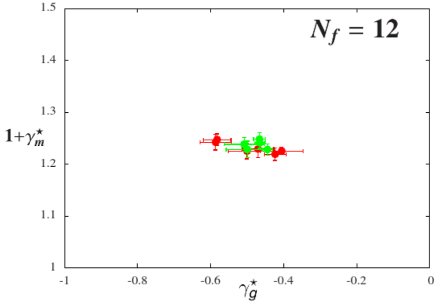
# Combined finite-size scaling analyses for $N_F = 12$

Approximate $M_H L \approx f_H(x)\left[1 + c_g m^{\omega}\right]$

$\longrightarrow$ consistent $\gamma_m^\star$ from all observables and $\beta_F$

Combined analyses of multiple data sets better constrain $\gamma_m^\star$ and $\gamma_g^\star$




Result from green points: $\gamma_m^\star = 0.235(15)$ and $\gamma_g^\star \simeq -0.5$

# Baryon anomalous dim. for partial compositeness

SM fermions $q$ couple linearly to $\mathcal{O}_q \sim \psi\psi\psi$ of new strong dynamics

$$\longrightarrow m_q \sim v \left(\frac{\text{TeV}}{\Lambda_F}\right)^{4-2\gamma_3}$$

with $\gamma_3 = \frac{9}{2} - \text{d}[\psi\psi\psi]$

Figure omitted to avoid weird pdf problem

Large mass hierarchy $\longleftrightarrow$ $\mathcal{O}(1)$ anomalous dimensions

Example: With $\Lambda_F = 10^{10}$ TeV, $\mathcal{O}$(MeV) quarks need $\gamma_3 \approx 1.75$
$\mathcal{O}$(GeV) quarks need $\gamma_3 \approx 1.9$

Compute $\gamma_{\mathcal{O}} = -\dfrac{d \log Z_{\mathcal{O}}(\mu)}{d \log \mu}$,

$Z_{\mathcal{O}}(\mu)$ from standard lattice RI/MOM non-perturbative renormalization

# Baryon anomalous dim. for partial compositeness

Figure omitted to avoid weird pdf problem

Compute $\gamma_{\mathcal{O}} = -\frac{d \log Z_{\mathcal{O}}(\mu)}{d \log \mu}$,

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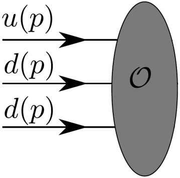
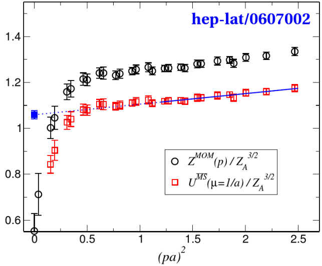


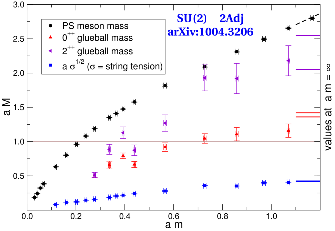

$\rightsquigarrow$

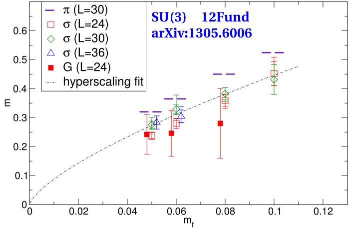


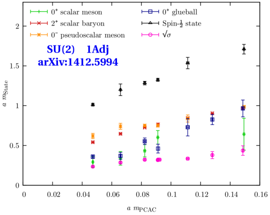

$N_F = 10, 12$ DWF pilot studies starting, re-using $\beta$ function work

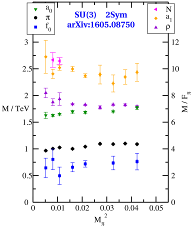
# Light scalars from beyond-QCD lattice calculations

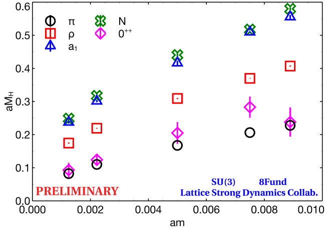
All near-conformal lattice studies so far observe light singlet scalar qualitatively different from QCD



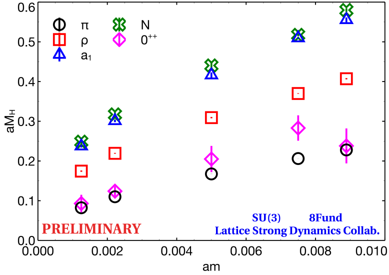



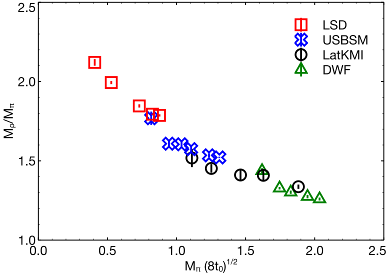



# Light scalar in 8-flavor SU(3) spectrum



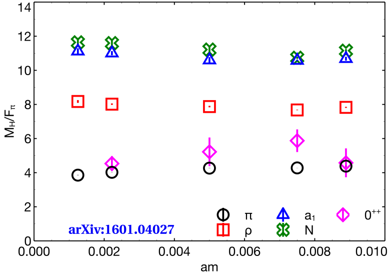



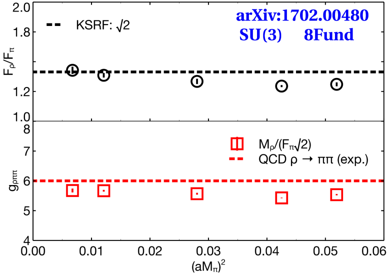

Flavor-singlet scalar degenerate with pseudo-Goldstones
down to lightest masses that fit into $64^3 \times 128$ lattices

Both $M_S$ and $M_P$ less than half the vector mass $M_V$,
hierarchy growing as we approach the chiral limit
$\longrightarrow$ qualitatively different from QCD

Controlled chiral extrapolations need EFT that includes scalar…

# Vector resonance generically QCD-like




Without EFT, roughly constant ratio $M_V/F_P \simeq 8 \;\rightsquigarrow\; M_V \simeq 2\text{ TeV}/\sqrt{\xi}$

[ NB: expect $M_P/F_P \to 0$ in chiral limit! ]

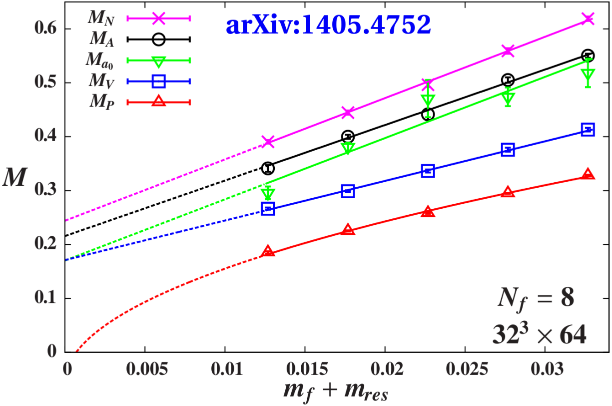
We measure $F_V \approx F_P\sqrt{2}$ (KSRF relation, suggesting vector domin.)

Applying second KSRF relation $g_{VPP} \approx M_V/(F_P\sqrt{2})$

$$\longrightarrow \text{vector width } \Gamma_V \approx \frac{g_{VPP}^2 M_V}{48\pi} \simeq 450 \text{ GeV — hard to see at LHC}$$

# QCD-like non-singlet scalar $a_0$ for $N_F = 8$

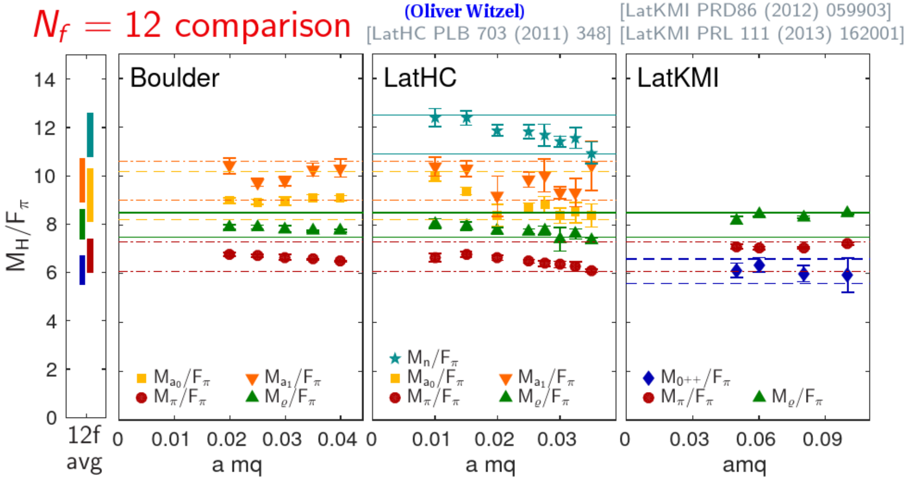
May be relevant for holographic approaches…




Earlier work with domain wall fermions farther from chiral limit

$\longrightarrow$ non-singlet scalar $a_0$ heavier than vector, $M_{a_0} \gtrsim M_V$

# QCD-like non-singlet scalar $a_0$ for $N_F = 12$




Staggered $N_F = 12$ results also show $M_{a_0} \gtrsim M_V$

Analyses complicated by staggered spin–flavor mixing

# Work in progress: Constraining EFT

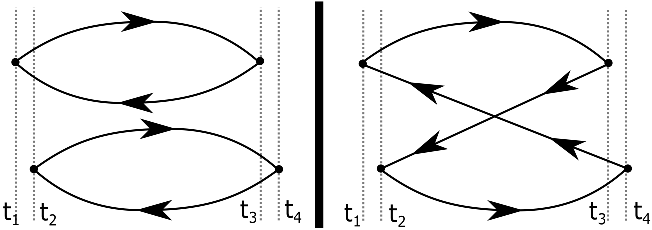
There are many candidate EFTs that include PNGBs + light scalar

(linear $\sigma$ model; Goldberger–Gristein–Skiba; Soto–Talavera–Tarrus; Matsuzaki–Yamawaki; Golterman–Shamir; Hansen–Langaeble–Sannino; Appelquist–Ingoldby–Piai)

### Need lattice computations of more observables to test EFTs

Now computing $2 \to 2$ elastic scattering of PNGBs & scalar, scalar form factor of PNGB

Subsequent step: Analog of $\pi K$ scattering in mass-split system

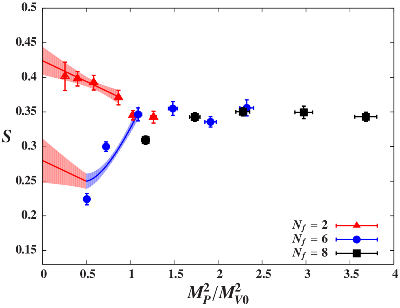


# $S$ parameter on the lattice

$$\mathcal{L}_\chi \supset \frac{\alpha_1}{2} g_1 g_2 B_{\mu\nu} \mathrm{Tr}\left[U \tau_3 U^\dagger W^{\mu\nu}\right] \longrightarrow$$


Lattice vacuum polarization calculation provides $S = -16\pi^2 \alpha_1$

Non-zero masses and chiral extrapolation needed
to avoid sensitivity to finite lattice volume




$S = 0.42(2)$ for $N_F = 2$
matches scaled-up QCD

Larger $N_F \longrightarrow$ significant reduction

Extrapolation to correct zero-mass limit
becomes more challenging

# Vacuum polarization from current correlator

$$S = 4\pi N_D \lim_{Q^2\to 0} \frac{d}{dQ^2} \Pi_{V-A}(Q^2) - \Delta S_{SM}(M_H)$$


$$\Pi^{\mu\nu}_{V-A}(Q) = Z \sum_x e^{iQ\cdot(x+\widehat{\mu}/2)} \mathrm{Tr}\left[\left\langle \mathcal{V}^{\mu a}(x) V^{\nu b}(0)\right\rangle - \left\langle \mathcal{A}^{\mu a}(x) A^{\nu b}(0)\right\rangle\right]$$

$$\Pi^{\mu\nu}(Q) = \left(\delta^{\mu\nu} - \frac{\widehat{Q}^{\mu}\widehat{Q}^{\nu}}{\widehat{Q}^2}\right) \Pi(Q^2) - \frac{\widehat{Q}^{\mu}\widehat{Q}^{\nu}}{\widehat{Q}^2} \Pi^L(Q^2) \qquad \widehat{Q} = 2\sin(Q/2)$$

- Renormalization constant $Z$ evaluated non-perturbatively
  Chiral symmetry of domain wall fermions $\Longrightarrow$ $Z = Z_A = Z_V$

  $Z = 0.85$ [2f]; 0.73 [6f]; 0.70 [8f]

- Conserved currents $\mathcal{V}$ and $\mathcal{A}$ ensure that lattice artifacts cancel

# Composite dark matter

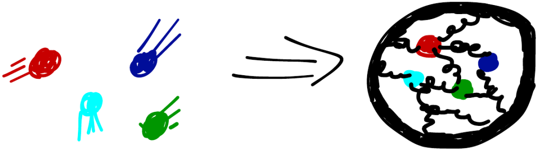
Many possibilities: (arXiv:1604.04627)

dark baryon, dark nuclei, dark pion, dark quarkonium, dark glueball...

**Example: Stealth Dark Matter** (arXiv:1503.04203, arXiv:1503.04205)

Deconfined charged fermions $\longrightarrow$ relic densnity

Confined SM-singlet dark baryon $\longrightarrow$ direct detection via form factors



For QCD-like SU(3) baryon, direct detection $\longrightarrow$ $M_{DM} \gtrsim 20$ TeV
due to leading magnetic moment interaction (arXiv:1301.1693)

# A lower bound for stealth dark matter

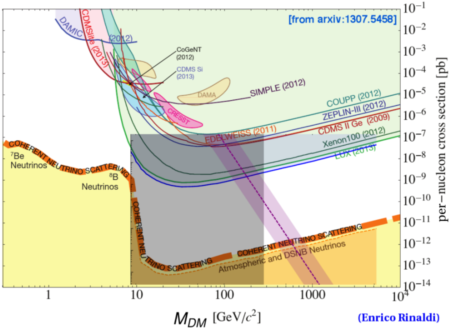
SU(4) bosonic baryons forbid leading magnetic moment
and sub-leading charge radius interactions in non-rel. EFT

EM polarizability is unavoidable — compute it on the lattice
$\longrightarrow$ lower bound on the direct detection rate

Nuclear cross section $\propto Z^4/A^2$,
these results specific to Xenon

Uncertainties dominated
by nuclear matrix element

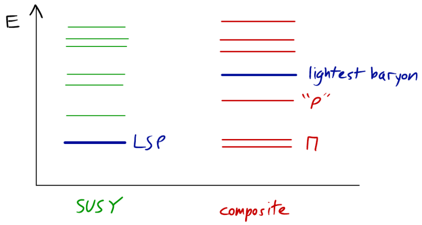
Shaded region is complementary
constraint from particle colliders

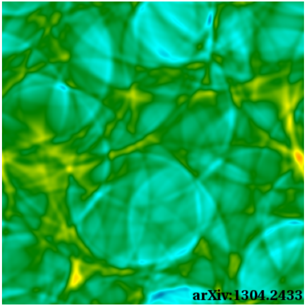


# Future plans: Colliders and gravitational waves

Other composite dark-sector states
can be discovered at colliders

Additional lattice input can help
predict production and decays




Confinement transition in early universe
may produce gravitational waves

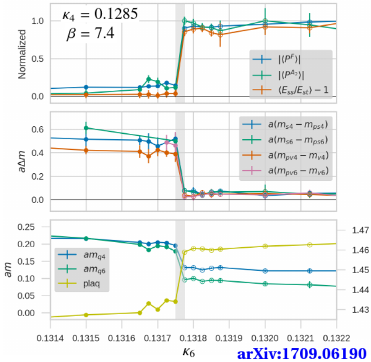
First-order transition $\longrightarrow$ colliding bubbles

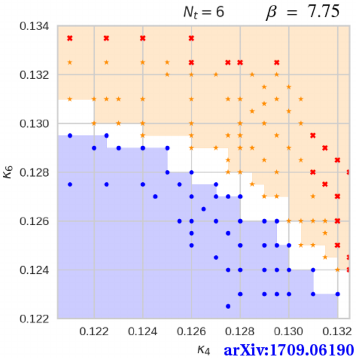
Lattice calculations needed
to predict properties of transition

# Multi-rep finite-temperature phase diagram

SU(4) gauge theory with $N_4 = 2$ fund. and $N_6 = 2$ two-index-symm.

Step towards composite Higgs model with $N_4 = 3$ and $N_6 = 2.5$

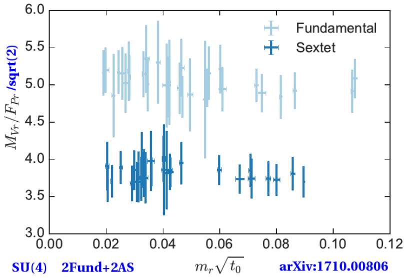


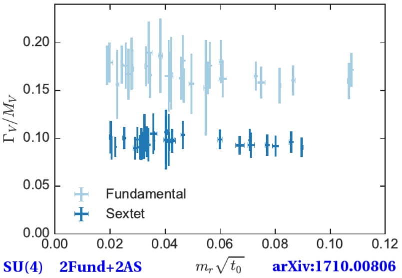




Simultaneous first-order chiral/deconfinement transitions for both reps

# Multi-rep mesonic spectrum

Looks broadly consistent with large-$N$ rescalings of QCD




**Left:** $M_V/F_P \sim 8\sqrt{\frac{3}{4}}\frac{1}{\sqrt{2}} \approx 4.9$

**Right:** Narrower vector resonance widths expected for larger $N$

# Recap: An exciting time for lattice gauge theory

Lattice gauge theory is a broadly applicable tool
to study strongly coupled systems and BSM physics

Exploring generic features of representative systems beyond QCD

- $\beta$ functions and anomalous dimensions
- Light scalar from near-conformal dynamics
- Low-energy constants including $S$ parameter
- Composite dark matter and more…

Thank you!

# Recap: An exciting time for lattice gauge theory

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**Thank you!**

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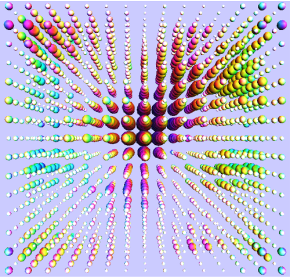
# Backup: Hybrid Monte Carlo (HMC) algorithm

Goal: Sample field configurations $U$ with probability $\frac{1}{\mathcal{Z}} e^{-S[U]}$



(Image credit: Claudio Rebbi)

HMC is Markov process based on
Metropolis–Rosenbluth–Teller

Fermions $\longrightarrow$ extensive action computation

$\Longrightarrow$ Global updates
using fictitious molecular dynamics

1. Introduce fictitious "MD time" $\tau$
   and stochastic canonical momenta for fields
2. Inexact MD evolution along trajectory in $\tau$
   $\longrightarrow$ new four-dimensional field configuration
3. Accept/reject test on MD discretization error

# Backup: Lattice QCD for BSM

High-precision non-perturbative QCD calculations
reduce uncertainties and help resolve potential new physics

- Hadronic matrix elements & form factors for flavor physics
  Sub-percent precision for easiest observables (arXiv:1607.00299)

- Hadronic contributions to $(g-2)_\mu$ (arXiv:1311.2198)
  Targeting $\sim$0.1% precision for vac. pol., $\sim$10% for light-by-light

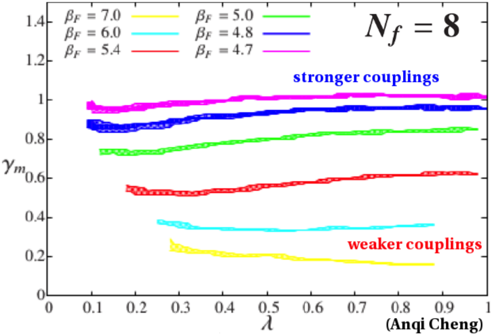
- $m_c$, $m_b$ and $\alpha_s(m_Z)$ to $\sim$0.1% for Higgs couplings (arXiv:1404.0319)

- High-temp. topological suscept. for axion DM (arXiv:1606.07494)

- Nucleon electric dipole moment, form factors (arXiv:1701.07792)

# Backup: $\gamma_{\text{eff}}(\lambda)$ from eigenmodes for $N_F = 8$

Fit $\nu(\lambda) \propto \lambda^{1+\alpha}$ in small range of $\lambda$ $\longrightarrow$ $1 + \gamma_{\text{eff}}(\lambda) = \dfrac{4}{1 + \alpha(\lambda)}$




$\nu(\lambda)$ computed stochastically

Include fit ranges in error bands

Multiple $L^4$ volumes overlaid,
$L$-sensitive data dropped

All systems have $\rho(0) = 0$

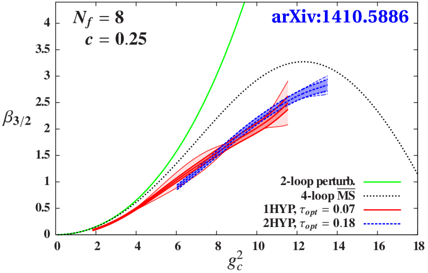
Appears to evolve slowly across wide range of scales,
qualitatively different from $N_F = 12$ and QCD-like $N_F = 4$

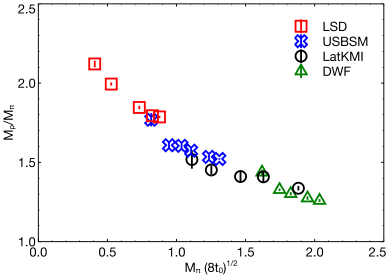
# Backup:
# Lattice Strong Dynamics Collaboration

| | |
|---|---|
| Argonne | Xiao-Yong Jin, James Osborn |
| Bern | DS |
| Boston | Rich Brower, Claudio Rebbi, Evan Weinberg |
| Colorado | Anna Hasenfratz, Ethan Neil, Oliver Witzel |
| UC Davis | Joseph Kiskis |
| Livermore | Pavlos Vranas |
| Oregon | Graham Kribs |
| RBRC | Enrico Rinaldi |
| Yale | Thomas Appelquist, George Fleming, Andrew Gasbarro |

Exploring the range of possible phenomena
in strongly coupled gauge theories

# Backup: 8-flavor SU(3) infrared dynamics




- $\beta$ function monotonic up to fairly strong $g^2 \sim 14$

  No sign of approach towards conformal IR fixed point $[\beta(g_\star^2) = 0]$

- Ratio $M_V/M_P$ increases monotonically as masses decrease

  as expected for spontaneous chiral symmetry breaking (S$\chi$SB)

  Mass-deformed conformal hyperscaling predicts constant ratio

Strengthen conclusion by matching to low-energy EFT

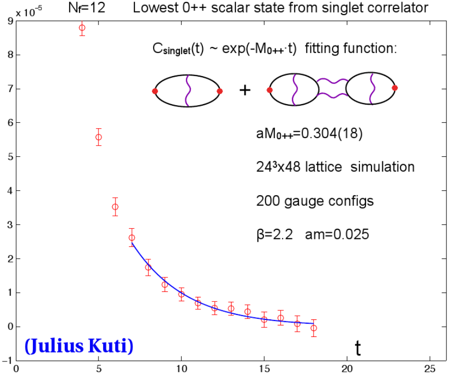
$\longrightarrow$ must go beyond QCD-like $\chi$PT to include light scalar...

# Backup: Technical challenge for scalar on lattice

Only new strong sector included in the lattice calculations

$\Longrightarrow$ flavor-singlet scalar mixes with the vacuum

Leads to noisy data and relatively large uncertainties

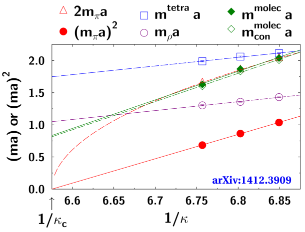


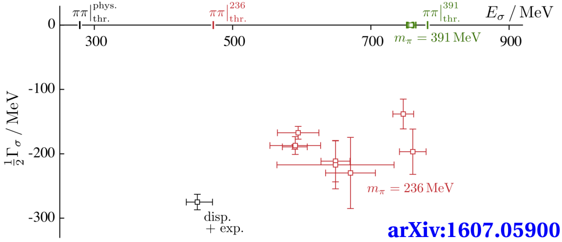

Fermion propagator computation relatively expensive

"Disconnected diagrams" formally need propagators at all $L^4$ sites

In practice estimate stochastically to control computational costs

# Backup: Isosinglet scalar in QCD spectrum




Lattice QCD $\longrightarrow$ isosinglet scalar much heavier than pion

Generally $M_S \gtrsim 2M_P$ $\longrightarrow$ $M_S > M_V$ for heavy quarks

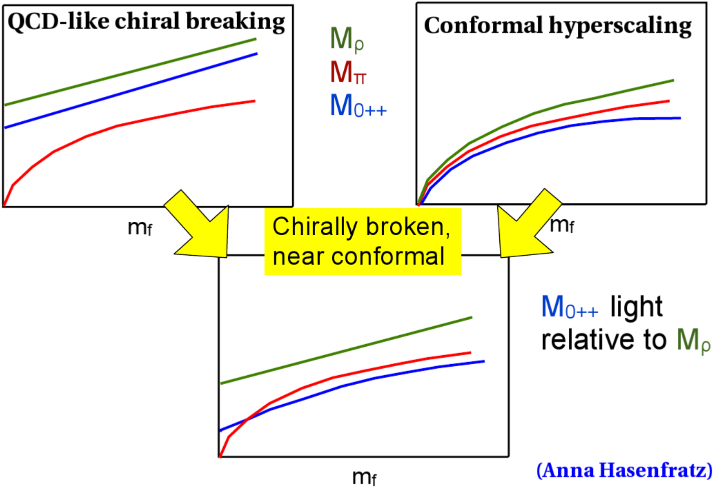
For a large range of quark masses $m$

it mixes significantly with two-pion scattering states

# Backup: Qualitative picture of light scalar

Light scalar likely related to near-conformal dynamics

$\longrightarrow$ possibly dilaton, PNGB of approximate scale symmetry?




$M_{0++}$ light relative to $M_\rho$

**(Anna Hasenfratz)**

# Backup: $2 \to 2$ elastic scattering on the lattice

Measure both $E_{PP}$ and $M_P \longrightarrow k = \sqrt{(E_{PP}/2)^2 - M_P^2}$

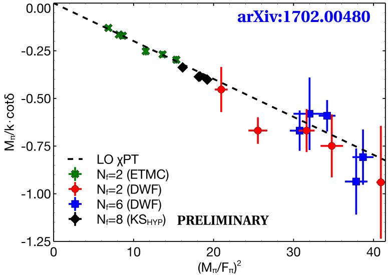
s-wave scattering phase shift: $\cot\delta_0(k) = \frac{1}{\pi k L}\, S\left(\frac{k^2 L^2}{4\pi}\right)$

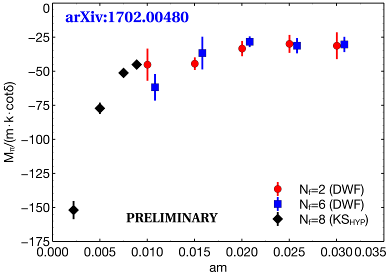
with regularized $\zeta$ function $S(\eta) = \sum_{j\neq 0}^{\Lambda} \frac{1}{j^2 - \eta} - 4\pi\Lambda$

Effective range expansion:

$$k\cot\delta_0(k) = \frac{1}{a_{PP}} + \frac{1}{2} M_P^2 r_{PP} \left(\frac{k^2}{M_P^2}\right) + \mathcal{O}\left(\frac{k^4}{M_P^4}\right)$$

# Backup: Initial 2 $\to$ 2 elastic scattering results



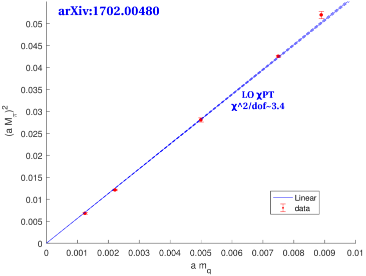

First looking at analog of QCD $\pi\pi$ scattering in $I = 2$ channel
(simplest case with no fermion-line-disconnected diagrams)

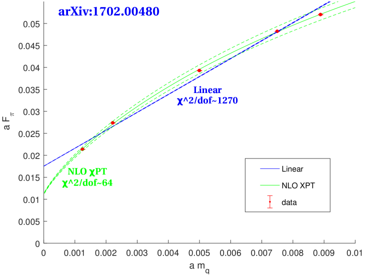
Simplest observable is scattering length $a_{PP} \approx 1/(k \cot\delta)$

$M_P a_{PP}$ vs. $M_P^2/F_P^2$ curiously close to leading-order $\chi$PT prediction

Dividing by fermion mass $m$ reveals expected tension with $\chi$PT
which predicts $M_P a_{PP}/m =$ const. at LO and involves 8 LECs at NLO

# Backup: 8f chiral perturbation theory ($\chi$PT) fits




In addition to omitting the light scalar

$\chi$PT also suffers from large expansion parameter

$$5.8 \leq \frac{2N_F Bm}{16\pi^2 F^2} \leq 41.3 \quad \text{for} \quad 0.00125 \leq m \leq 0.00889$$

Big ($\sim 50\sigma$) shift in $F$ from linear extrapolation vs. NLO $\chi$PT

Fit quality is not good, especially for NLO joint fit with $\chi^2/\text{d.o.f.} > 10^4$

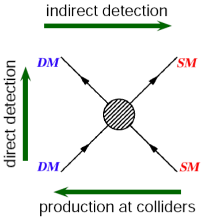
# Backup: NLO chiral perturbation theory formulas

$$M_P^2 = 2Bm\left[1 + \frac{2N_F Bm}{16\pi^2 F^2}\left\{128\pi^2\left(2L_6^r - L_4^r + \frac{2L_8^r - L_5^r}{N_F}\right) + \frac{\log\left(2Bm/\mu^2\right)}{N_F^2}\right\}\right]$$

$$F_P = F\left[1 + \frac{2N_F Bm}{16\pi^2 F^2}\left\{64\pi^2\left(L_4^r + \frac{L_5^r}{N_F}\right) - \frac{1}{2}\log(2Bm/\mu^2)\right\}\right]$$

$$\begin{aligned} M_P a_{PP} = \frac{-2Bm}{16\pi F^2}\left[1 + \frac{2N_F Bm}{16\pi^2 F^2}\left\{-256\pi^2\left(\left[1 - \frac{2}{N_F}\right](L_4^r - L_6^r)\right.\right.\right. \\ \left. + \frac{L_0^r + 2L_1^r + 2L_2^r + L_3^r}{N_F}\right) - 2\frac{N_F - 1}{N_F^3} \\ \left.\left. + \frac{2 - N_F + 2N_F^2 + N_F^3}{N_F^3}\log\left(2Bm/\mu^2\right)\right\}\right] \end{aligned}$$

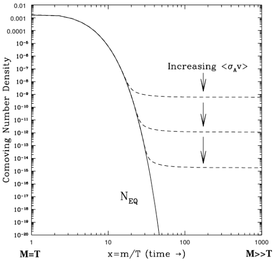
# Backup: Thermal freeze-out for relic density




$T \gtrsim M_{DM}$: DM $\longleftrightarrow$ SM
Thermal equilibrium

$T \lesssim M_{DM}$: DM $\longrightarrow$ SM
Rapid depletion of $\Omega_{DM}$

Hubble expansion $\longrightarrow$ dilution
leads to freeze-out




Requires coupling between ordinary matter and dark matter

Mass and coupling of pure thermal relic are related: $\dfrac{M_{DM}}{100\text{ GeV}} \sim 200\alpha$

With strong $\alpha \sim 16$, 'natural' mass scale is $M_{DM} \sim 300$ TeV

At smaller masses $M_{DM} \gtrsim 1$ TeV
thermal relic could be just part of total relic density

# Backup: Two roads to natural asymmetric dark matter

**Idea:** Dark matter relic density related to baryon asymmetry

$$\Omega_D \approx 5\Omega_B$$
$$\implies M_D n_D \approx 5 M_B n_B$$

- $n_D \sim n_B \quad \implies \quad M_D \sim 5M_B \approx 5$ GeV
  High-dim. interactions relate baryon# and DM# violation

- $M_D \gg M_B \quad \implies \quad n_B \gg n_D \sim \exp\left[-M_D/T_s\right] \qquad T_s \sim 200$ GeV
  EW sphaleron processes above $T_s$ distribute asymmetries

Both require coupling between ordinary matter and dark matter

# Backup: Composite dark matter interactions

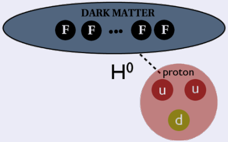
## Photon exchange via electromagnetic form factors

Interactions suppressed by powers of confinement scale $\Lambda \sim M_{DM}$

- **Dimension 5:** Magnetic moment $\longrightarrow$ $\left(\overline{\psi}\sigma^{\mu\nu}\psi\right) F_{\mu\nu}/\Lambda$
- **Dimension 6:** Charge radius $\longrightarrow$ $\left(\overline{\psi}\psi\right) v_\mu \partial_\nu F_{\mu\nu}/\Lambda^2$
- **Dimension 7:** Polarizability $\longrightarrow$ $\left(\overline{\psi}\psi\right) F^{\mu\nu} F_{\mu\nu}/\Lambda^3$

## Higgs exchange via scalar form factors

Effective Higgs interaction of composite DM
needed for correct Big Bang nucleosynthesis



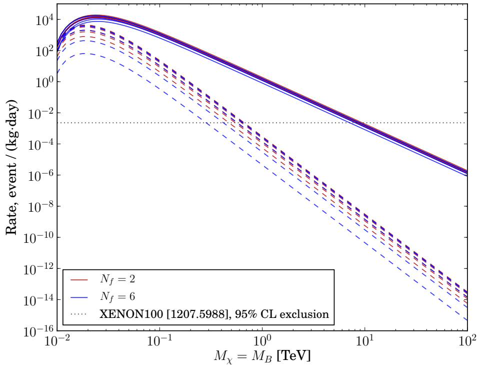

Higgs couples through $\langle B \left| m_\psi \overline{\psi}\psi \right| B\rangle$ ($\sigma$ terms)

All form factors arise non-perturbatively $\Longrightarrow$ lattice calculations

# Backup: SU(3) direct detection constraints

Solid lines are predictions for total number of events XENON100 would observe for SU(3) model with dark baryon mass $M_B$

Dashed lines are subleading charge radius contribution suppressed $\sim 1/M_B^2$ relative to magnetic moment contribution

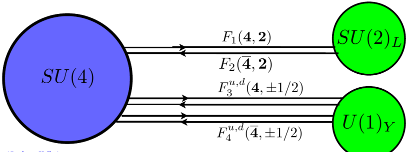



XENON100 results

(arXiv:1207.5988)
exclude $M_B \lesssim 10$ TeV

SU($N$) with even $N \geq 4$ forbids mag. moment. . .

# Backup: Stealth dark matter model details




(Graham Kribs)

| Field | $SU(N_D)$ | $(SU(2)_L,\ Y)$ | $Q$ |
|---|---|---|---|
| $F_1 = \begin{pmatrix} F_1^u \\ F_1^d \end{pmatrix}$ | $\mathbf{N}$ | $(\mathbf{2},\ 0)$ | $\begin{pmatrix} +1/2 \\ -1/2 \end{pmatrix}$ |
| $F_2 = \begin{pmatrix} F_2^u \\ F_2^d \end{pmatrix}$ | $\bar{\mathbf{N}}$ | $(\mathbf{2},\ 0)$ | $\begin{pmatrix} +1/2 \\ -1/2 \end{pmatrix}$ |
| $F_3^u$ | $\mathbf{N}$ | $(\mathbf{1}, +1/2)$ | $+1/2$ |
| $F_3^d$ | $\mathbf{N}$ | $(\mathbf{1}, -1/2)$ | $-1/2$ |
| $F_4^u$ | $\bar{\mathbf{N}}$ | $(\mathbf{1}, +1/2)$ | $+1/2$ |
| $F_4^d$ | $\bar{\mathbf{N}}$ | $(\mathbf{1}, -1/2)$ | $-1/2$ |

Mass terms $\sim m_V\,(F_1 F_2 + F_3 F_4) + y\left(F_1 \cdot H F_4 + F_2 \cdot H^\dagger F_3\right) + \text{h.c.}$

Both vector-like masses $m_V$ and Higgs couplings $y$ are **required**

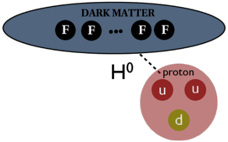
- Higgs couplings ensure rapid meson decay in early universe
- Vector-like masses avoid bounds on direct detection via Higgs exchange

# Backup: Effective Higgs interaction

With $M_H = 125$ GeV, Higgs exchange may dominate

spin-independent direct detection cross section

$$\sigma_H^{(SI)} \propto \left| \frac{\mu_{B,N}}{M_H^2} \; y_\psi \langle B | \overline{\psi}\psi | B \rangle \; y_q \langle N | \overline{q} q | N \rangle \right|^2$$




For quarks $y_q = \dfrac{m_q}{v} \implies y_q \langle N | \overline{q} q | N \rangle \propto \dfrac{M_N}{v} \dfrac{\langle N | m_q \overline{q} q | N \rangle}{M_N}$

For dark constituent fermions $\psi$

there is an additional model parameter, $y_q = \alpha \dfrac{m_\psi}{v}$

In both cases the scalar form factor is most easily determined

using the Feynman–Hellmann theorem $\dfrac{\langle B | m_\psi \overline{\psi}\psi | B \rangle}{M_B} = \dfrac{m_\psi}{M_B} \dfrac{\partial M_B}{\partial m_\psi}$

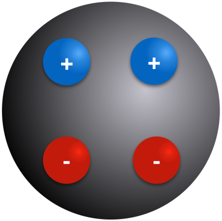
# Backup: Stealth dark matter EM form factors

Lightest SU(4) composite dark baryon

Scalar particle $\longrightarrow$ no magnetic moment

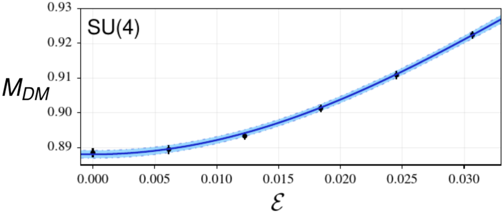
+/- charge symmetry $\longrightarrow$ no charge radius

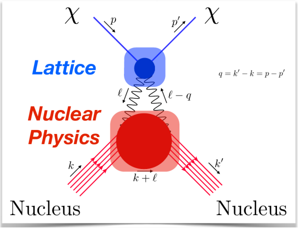
Higgs exchange can be negligibly small




Polarizability places lower bound on direct-detection cross section

Compute on lattice as dependence of $M_{DM}$ on external field $\mathcal{E}$



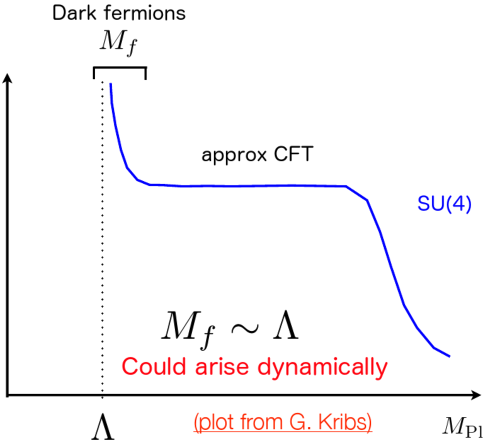



# Backup: Stealth dark matter mass scales

Lattice calculations have focused on $m_\psi \simeq \Lambda_D$,
the regime where analytic estimates are least reliable

This mass scale has
some theoretical motivation

In addition,
collider constraints tighten
as mass decreases

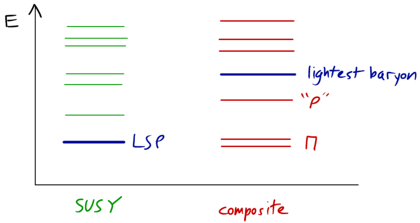


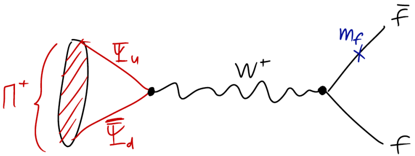
# Backup: Stealth dark matter collider detection

Spectrum significantly different
from MSSM-inspired models

Very little missing $E_T$ at colliders

Main constraints from
much lighter **charged** "Π" states




Rapid Π decays with $\Gamma \propto m_f^2$

Best current constraints
recast stau searches at LEP

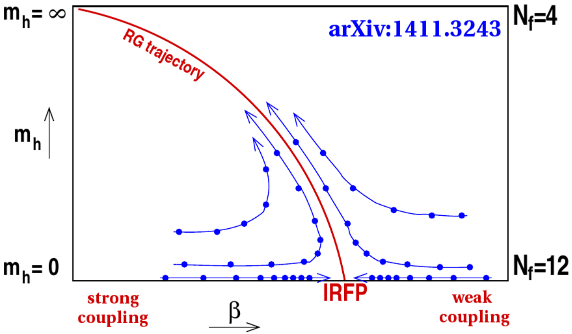
LHC can also search for $t\bar{b} + \bar{t}b$
from $\Pi^+\Pi^-$ Drell–Yan production

# Backup: Philosophy of mixed-mass approach

$N_F = N_\ell + N_h$ fermions, light $m_\ell \to 0$ at fixed $m_h > 0$

Allows large $N_F$ for approximate conformality
without introducing extra Goldstones

Reducing $m_h$ extends the range of scales
over which theory is governed by conformal fixed point




Real-space RG flow lines
(from UV to IR)

$\gamma_m$ above considered
strong-coupling side
$\longrightarrow$ "backward" flow