Hierarchy of Scales in Strongly-Coupled Theories

Maurizio Piai (Swansea University)

D.Elander and MP, arXiv:1703.10158,
D.Elander and MP, arXiv:1703.09205.
AND REFERENCES THEREIN

Outline

- Statement of the problem: big and small hierarchy problems.
- Introduction I: Higgs particle as a dilaton
- Introduction II: gauge/gravity dualities and the baryonic branch of KS.
- Field theory: the KS quiver and its properties—brief review of long (hi)story
- Supergravity: PT ansatz, the baryonic branch solutions—very brief review

  A. Butti, M. Grana, R. Minasian, M. Petrini, A. Zaffaroni, hep-th/0412187
- Spectrum of glueballs (our contribution)
- Outlook

Statement of the Problem

- Weakly-coupled (Higgs field) models of EWSB are UV-incomplete, and as EFT they contain unprotected operators: BIG HIERARCHY PROBLEM.
- Strong-coupling solution: replace Higgs sector with new interactions and fields, that UV-complete the theory (technicolor, composite Higgs, little Higgs...)
- Calculability problems; NDA (EFT) shows QCD-like theories not phenomenologically viable. Can we compute/make predictions beyond NDA?
- Phenomenological problems; QCD-like theory cannot work (precision EW tests, top mass, PNGB proliferation, FCNC, Higgs particle…):LITTLE HIERARCHY PROBLEM.

Statement of the Problem—cartoon

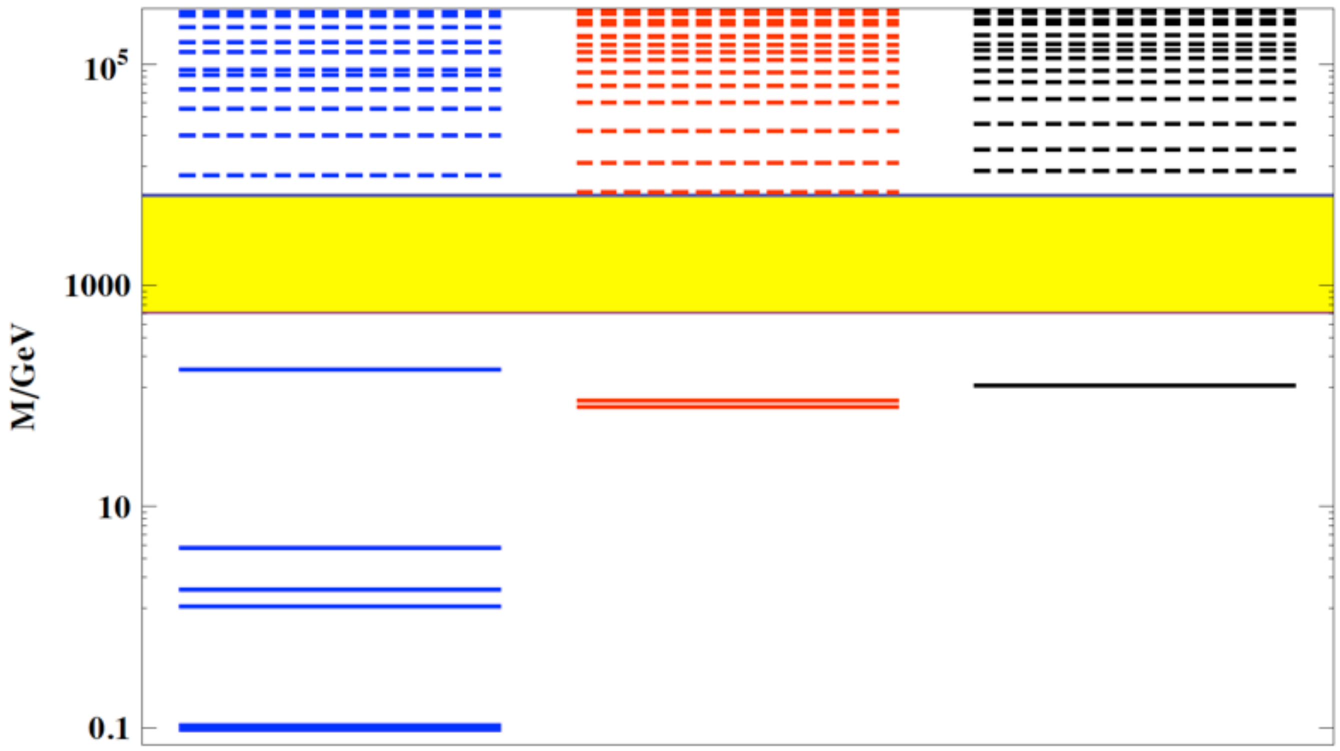

Figure 1. *The mass spectrum of SM particles (continuous lines), compared to the range of current exclusions from LHC direct searches for exotics (shaded region) [2] and to the spectrum of a generic, hypothetic strongly-coupled new physics theory (dashed lines) with new states heavy enough to avoid current direct bounds. Fermions are rendered in blue, vectors in red and scalars in black.*

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- Calculability problems; NDA (EFT) shows QCD-like theories not phenomenologically viable. Can we compute/make predictions beyond NDA?
- Phenomenological problems; QCD-like theory cannot work (precision EW tests, top mass, PNGB proliferation, FCNC, Higgs particle...): LITTLE HIERARCHY PROBLEM.
- Goal: find a strongly-coupled model that has dynamical features very different from QCD. Dynamics is multi-scale.
- Tool: gauge/gravity dualities.

Higgs particle as a Dilaton

MINIMAL STANDARD MODEL

- Gauge bosons kinetic terms

$$\mathcal{L}_1 = -\frac{1}{2}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu} + \cdots ,$$

- Fermions kinetic terms

$$\mathcal{L}_{1/2} = \bar{\psi}\, i\not{D}\, \psi + \cdots ,$$

- Scalar kinetic term

$$\mathcal{L}_0 = (D_\mu H)^\dagger D^\mu H$$

- Yukawa couplings

$$\mathcal{L}_y = -y\, \bar{\psi}_L\, H\, \psi_R + \cdots ,$$

- Scalar potential

$$\mathcal{L} = -\mathcal{V} = -\mu^2 H^\dagger H - \lambda \left(H^\dagger H\right)^2 .$$

Minimization

- Vacuum Expectation Value (VEV)

$$\langle H^\dagger H \rangle = -\frac{\mu^2}{2\lambda} \equiv \frac{v^2}{2} ,$$

- Physical Higgs mass

$$H = \frac{v+h}{\sqrt{2}} \longrightarrow m_h^2 = -2\mu^2 = 2\lambda v^2 .$$

Classical scaling dimensions

- Action

$$\mathcal{S} = \int \mathrm{d}^4x\, \mathcal{L} \longrightarrow [\mathcal{L}] = [x]^{-4}$$

- Bosons

$$[A_\mu] = [x]^{-1} \longrightarrow [F_{\mu\nu}] = [\partial_\mu A_\nu] = [x]^{-2} ,$$
$$[H] = [x]^{-1} \longrightarrow [D_\mu H] = [\partial_\mu H] = [x]^{-2} ,$$

- Fermions

$$[\psi] = [x]^{-3/2} \longrightarrow [D_\mu \psi] = [\partial_\mu \psi] = [x]^{-5/2} ,$$

Conformal Symmetry Breaking

- **Explicit**: the μ^2 term is the **ONLY** term in the Lagrangian that breaks dilatation symmetry at the classical level.
- **Spontaneous**: the VEV v^2 breaks the symmetry in the vacuum.
- **Higgs as a Dilaton**: taking the limit $\lambda \to 0$ ($\mu^2 \to 0$) while keeping v^2 fixed, implies $m_h^2 \to 0$. The Higgs is the dilaton, the pseudo-Goldstone boson associated with global scaling invariance.

Higgs/Dilaton Couplings

- Classically, the SM Higgs particle is a (pseudo-)dilaton.

- Coupling to stress-energy tensor yields coupling controlled by the masses:

$$\begin{aligned}\mathcal{L} \;=\; & 2\frac{h}{v}\, m_W^2\, W^{\mu\,+} W_\mu^- \\ & + \frac{h}{v}\, m_Z^2\, Z^\mu Z_\mu \\ & - \frac{h}{v}\, m_\psi\, \bar{\psi}\psi\; \cdots\end{aligned}$$

- Huge predictive power: one parameter (the mass of the Higgs particle, breaking symmetry).

- Deviations come from quantum effects (coupling to gluons and photons...) and/or from higher-order operators (new physics at TeV scale…), or other new particles, or from decay constant being larger than EW scale…

- General question: in your favourite extension of the Standard Model, is there a dilaton? If so, it will resemble a light Higgs particle.

More general QFT question:

irrespectively of SM, EWSB, phenomenological and model-building considerations,

is there a calculable example of strongly-coupled QFT with a light dilaton, and hierarchy of scales, in the spectrum?

Gauge-Gravity Dualities

- Some special QFT admits equivalent description as higher-dimensional theory coupled to gravity. Strong-Weak Duality. AdS/CFT correspondence, generalised to gauge/gravity correspondence.

J.M.Maldacena hep-th/9711200S.
S. Gubser, I. R. Klebanov and A. M. Polyakov, hep-th/9802109
E. Witten, hep-th/9802150

- Dictionary relating the two:

$$\begin{aligned} \text{bulk field}\,\Phi &\leftrightarrow \mathcal{O}\,\text{operator} \\ \text{boundary value}\,\varphi_0 &\leftrightarrow J\,\text{source} \end{aligned}$$

K. Skenderis, hep-th/0209067

$$\int_{\Phi\to\varphi_0} \mathcal{D}\Phi\, e^{-\mathcal{S}[\Phi]} = \left\langle e^{-\int_{\partial M}\varphi_0\,\mathcal{O}} \right\rangle$$

- Weak-Strong duality: saddle-point approximation

$$\mathcal{S}[\Phi]_{\mathrm{on-shell}} = -W[\varphi_0] = -\log \mathcal{Z}[J]$$

- Most useful in large-N (classical), large ‘t Hooft (supergravity) regime of QFT (string theory).

- In this talk, top-down approach: we always start from complete string/sugra model in D=10,11.

Gauge-Gravity Dualities
Top-down approach

- Start from 10D superstring theory (Type IIB for example), consider supergravity limit.
- Write a general ansatz: internal 5D compact manifold with given symmetries, non-compact 5D.
- Perform KK reduction to 5D (obtain infinite number of 5D states, discrete spectrum).
- Choose subgroup of symmetries, and perform consistent truncation (keep only few 5D states).

- Write sigma-model with n scalars coupled to 5D gravity.
- Solve bulk equations for scalars and gravity, and identify physical meaning of integration constants.
- Fix background of interest (=choose and fix integration constants).

- Add boundaries in UV and IR, as regulators, and infer appropriate boundary conditions.
- Fluctuate 5D scalars and gravity.
- Rewrite fluctuations in gauge-invariant form and focus on physical degrees of freedom.
- Solve for scalar fluctuations and mass spectrum.
- Remove regulators (if possible), and obtain physical quantities of dual field theory.

- Lift to 10-dimensions.
- Study extended objects, probe strings (confinement), probe D-branes (chiral symmetry breaking)...

Gauge-Gravity Dualities
Bottom-up approach

- Write sigma-model with n scalars coupled to 5D gravity.
- Solve bulk equations for scalars and gravity, and identify physical meaning of integration constants.
- Fix background of interest (=choose and fix integration constants).

- Add boundaries in UV and IR, as regulators, and infer appropriate boundary conditions.
- Fluctuate 5D scalars and gravity.
- Rewrite fluctuations in gauge-invariant form and focus on physical degrees of freedom.
- Solve for scalar fluctuations and mass spectrum.
- Remove regulators (if possible), and obtain physical quantities of dual field theory.

Gauge-Gravity Dualities

- Bottom-up approach advantages: flexible model-building, calculation of masses accessible, rich phenomenology accessible, many different scenarios possible, and many different observables calculable.
- Bottom-up approach disadvantages: not a fundamental theory, not even a fully satisfactory EFT, provides incomplete description of confinement (no area law/flux tube).

- Top-down advantages: derived from fundamental string theory model (very rigid structure), clean field theory interpretation, confinement admits sensible description.
- Top-down disadvantages: model-building highly challenging, computing mass spectrum often hard, models are unrealistic phenomenologically.

- Complementarity of the two: use the former to do phenomenology, the latter to learn about field theory fundamental properties not accessible otherwise.

Gauge-Gravity Dualities Examples

- Maldacena AdS(5)xS(5) vs. N=4 SYM (CFT) — J.M.Maldacena hep-th/9711200
- Pilch-Warner (IR fixed point) — R.G. Leigh and M.J. Strassler, hep-th/9503121; K. Pilch and N.P. Warner, hep-th/0006066
- Witten (Confinement) — E.Witten, hep-th/9803131
- Klebanov-Strassler (Seiberg duality, confinement and gaugino condensation) — I.R. Klebanov and M.J Strassler, hep-th/0007191
- Maldacena-Nunez (Confinement, gaugino condensation, and Higgsing) — A.H.Chamseddine, M.S.Volkov, hep-th/9707176; J.M.Maldacena and C. Nunez, hep-th/0008001
- Sakai-Sugimoto (Chiral symmetry breaking) — T.Sakai and S.Sugimoto, hep-th/0412141
- Baryonic Branch of KS (Moduli space) — A. Butti, M. Grana, R. Minasian, M. Petrini and A. Zaffaroni, hep-th/0412187,

Many non-trivial features of strongly coupled models can be studied precisely, although within QFTs of marginal intrinsic interest for strict phenomenological purposes.

Can we use these techniques for physically relevant questions?

5D sigma-models (consistent truncation)

- Systematic way of constructing sugra backgrounds uses consistent truncation to 5D sigma-model (n scalars) coupled to gravity.

$$\begin{aligned} \mathcal{S} \;\equiv\; & \int \mathrm{d}^4x\mathrm{d}r \left\{ \sqrt{-g}\Theta\left[\frac{1}{4}R + \mathcal{L}_5(\Phi^a, \partial_M\Phi^a, g)\right] \right. \\ & + \sqrt{-\tilde{g}}\delta(r-r_1)\left[c_K K + \mathcal{L}_1(\Phi^a, \partial_\mu\Phi^a, \tilde{g})\right] \\ & \left. - \sqrt{-\tilde{g}}\delta(r-r_2)\left[c_K K + \mathcal{L}_2(\Phi^a, \partial_\mu\Phi^a, \tilde{g})\right] \right\} \end{aligned}$$

$$\mathrm{d}s_{1,4}^2 \equiv e^{2A}\eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu \;+\; \mathrm{d}r^2$$

$$\begin{aligned} \mathcal{L}_5 &\equiv -\frac{1}{2}G_{ab}g^{MN}\partial_M\Phi^a\partial_N\Phi^b - V(\Phi^a) \\ \mathcal{L}_1 &\equiv -\lambda_{(1)}(\Phi^a)\,, \\ \mathcal{L}_2 &\equiv -\lambda_{(2)}(\Phi^a)\,. \end{aligned}$$

- Bulk equations and boundary terms determine 5D background, lift to 10D known.

$$\bar{\Phi}''^a + 4A'\bar{\Phi}'^a + \mathcal{G}^a{}_{bc}\bar{\Phi}'^b\bar{\Phi}'^c - V^a = 0$$

$$6A'^2 + 3A'' = -G_{ab}\bar{\Phi}'^a\bar{\Phi}'^b - 2V,$$

$$6A'^2 = G_{ab}\bar{\Phi}'^a\bar{\Phi}'^b - 2V.$$

- First-order equations may exist:

$$V \;=\; \frac{1}{2}G^{ab}W_aW_b \;-\; \frac{4}{3}W^2 \qquad A' \;=\; -\frac{2}{3}W\,,$$

$$\bar{\Phi}'^{\,a} \;=\; G^{ab}W_b \;=\; W^a$$

- Lift to ten dimensions: given a complete solution in five dimensions, purely algebraic process yields whole supergravity solution. Notice: non-singular 10-dimensional solutions may be obtained from singular five-dimensional ones.

Computing the Spectrum of Scalar and Tensor Glueballs

- Given a background, one can study the spectrum of scalar fluctuations (systematic algorithmic procedure exists!), using gauge-invariant variables:

$$
\begin{aligned}
\mathfrak{a}^a &= \varphi^a - \frac{\bar{\Phi}'^a}{6A'}h, \\
\mathfrak{b} &= \nu - \frac{\partial_r(h/A')}{6}, \\
\mathfrak{c} &= e^{-2A}\partial_\mu \nu^\mu - \frac{e^{-2A}\Box h}{6A'} - \frac{1}{2}\partial_r H, \\
\mathfrak{d}^\mu &= e^{-2A}\Pi^\mu{}_\nu \nu^\nu - \partial_r \epsilon^\mu, \\
\mathfrak{e}^\mu{}_\nu &= h^{TT\mu}{}_\nu.
\end{aligned}
$$

Berg, Haack, Mueck hep-th/0507285

- Bulk equations and boundary terms known in general:

$$\left[\partial_r^2 + 4\partial_r A \partial_r + e^{-2A}m^2\right]\mathfrak{e}^\mu{}_\nu = 0$$

$$\left[\mathcal{D}_r^2 + 4A'\mathcal{D}_r + e^{-2A}\Box\right]\mathfrak{a}^a - \left[V^a{}_{|c} - \mathcal{R}^a{}_{bcd}\bar{\Phi}'^b\bar{\Phi}'^d + \frac{4(\bar{\Phi}'^a V_c + V^a\bar{\Phi}'_c)}{3A'} + \frac{16V\bar{\Phi}'^a\bar{\Phi}'_c}{9A'^2}\right]\mathfrak{a}^c = 0,$$

$$\partial_r \mathfrak{e}^\mu{}_\nu\Big|_{r=r_i} = 0$$

D. Elander, MP, arXiv:1010.1964

$$\left[\delta^a_b + e^{2A}\Box^{-1}\left(V^a - 4A'\Phi'^a - \lambda^a{}_{|c}\bar{\Phi}'^c\right)\frac{2\bar{\Phi}'_b}{3A'}\right]\mathcal{D}_r\mathfrak{a}^b\Big|_{r_i} =$$

$$\left[\lambda^a{}_{|b} + \frac{2\bar{\Phi}'^a\bar{\Phi}'_b}{3A'} + e^{2A}\Box^{-1}\frac{2}{3A'}\left(V^a - 4A'\bar{\Phi}'^a - \lambda^a{}_{|c}\bar{\Phi}'^c\right)\left(\frac{4V\bar{\Phi}'_b}{3A'} + V_b\right)\right]\mathfrak{a}^b\Big|_{r_i}$$

- Procedure: take your confining background, introduce UV and IR cutoffs (regulators!), solve bulk equations and apply boundary conditions, repeat by progressively removing the two cutoffs. If IR and UV are healthy, the cutoff effects will decouple.

Typical Examples: Yang-Mills-like spectra

D. Elander, A.Faedo, C.Hoyos, D. Mateos, MP, arXiv:1312.71.60

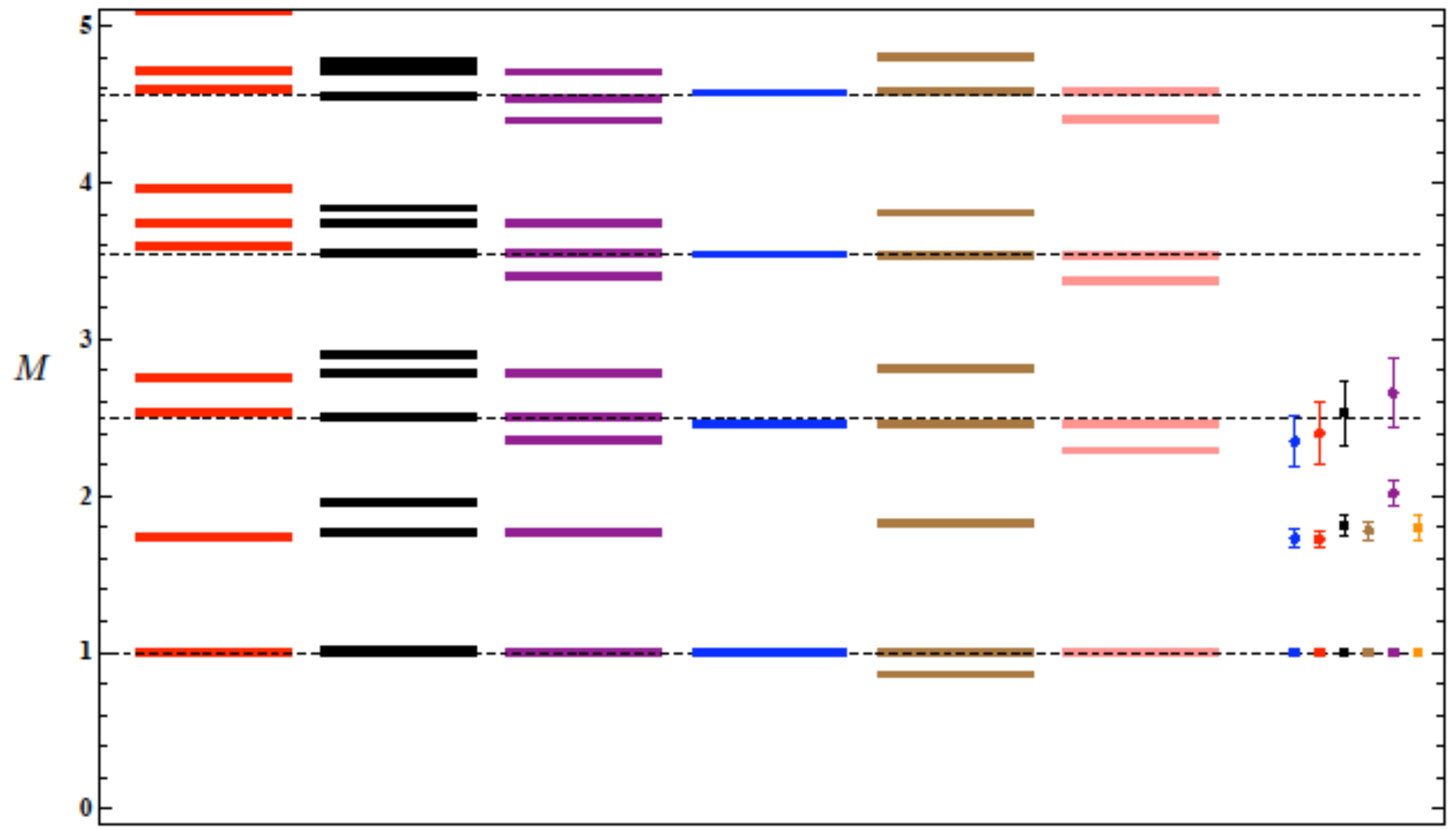


Figure 14. Spectrum of 0^{++} glueballs according to several different calculations. Left to right: the spectrum of the Witten model (the M-theory model at the UV fixed point) according to [45] (red) and according to our calculations in Section 4.3 (black), the spectrum obtained from the M-theory model at the IR, non-supersymmetric fixed-point, also discussed in Section 4.3 (purple), the spectrum obtained by considering the string model at either fixed point and truncating ϕ from the spectrum [56] (blue), the spectrum obtained from the string theory model at the UV, supersymmetric fixed point discussed in Section 3.3 (brown) and at the IR, non-supersymmetric fixed point, also discussed in Section 4.3 (pink). The normalisation of the spectra is discussed in the main text. The dotted lines, which lie at $M = \{1, 2.5, 3.54, 4.56\}$, highlight the subset of states that is common to all the models (within numerical precision). Finally, in the last column we report lattice results for $SU(N_c)$ Yang-Mills, with $N_c = 3, \ldots, 8$ (left to right), as discussed in the text.

[45]:R.C.Brower, S.D.Mathur,C-I. Tan , hep-th/0003115

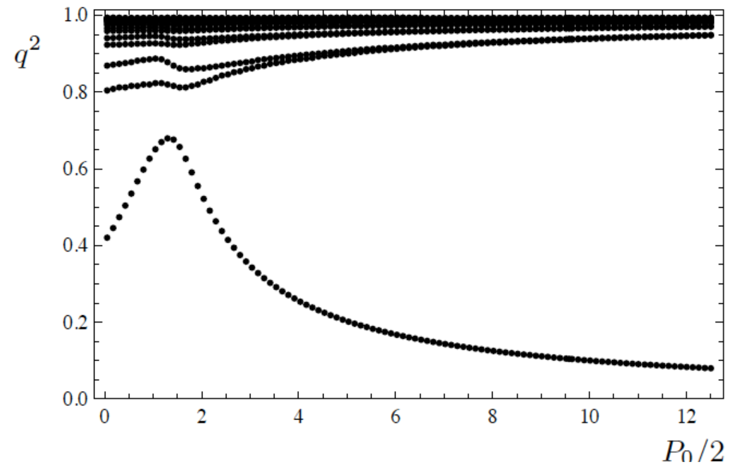
[56]:C-K Wen, H-X Yang, hep-th/0404152

B. Lucini M Panero, arXiv:1210.4997

Top-down dual backgrounds to ordinary, confining, strongly-coupled, four-dimensional theories exhibit spectra of glueballs that qualitatively resemble lattice results for Yang-Mills.

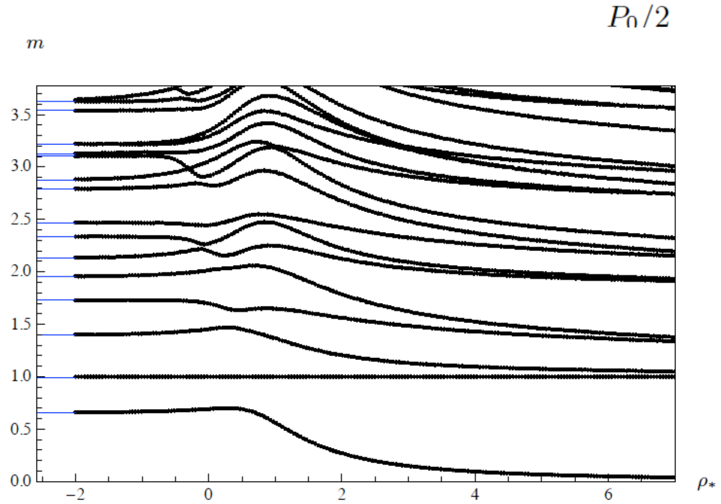
Light Dilaton
Earlier Examples

- Non-QCD-like models with walking dynamics (in the sense that coupling varies slowly).
- In CVMN system:



D.Elander, C.Nunez and MP, 0908.2808
D.Elander and MP, 1212.2600

- In KS system:



D.Elander, 1401.3412

- Parametrically light glueball (dilaton) emerges, when the walking region (x-axis here...) is long.
- Large VEV for dimension-6 operator in dual QFT language turned on: multi scale theory.
- Complicated, model-dependent spectrum appears starting from largish scale, above EWSB scale.

Light Dilaton
Earlier Examples

- QFT interpretation obscure: higher-dimensional operator (6-dimensional) develops VEV. Why?
- QFT interpretation obscure: UV-asymptotic behaviour of theory unclear.

GREAT PROMISE, BUT WITH
LIMITATIONS,
CAN WE DO BETTER/MORE?

- Sugra treatment incomplete: IR singularity in (10-dimensional) background. Good singularity?
- Sugra treatment incomplete: UV behaviour not asymptotically-AdS.

- Can we find example where these perplexing features do not arise? YES, the baryonic branch of KS.
- What is the price to pay? Technical difficulty in finding such examples and studying them.

The KS baryonic branch: Quiver QFT

- Susy theory in 4 dimensions. Field content known and dynamics understood, thanks to susy.

	$SU(M)$	$SU(M+N)$	$SU(2)_A$	$SU(2)_B$	$U(1)_B$	$U(1)_R$
A_α	M	$\overline{M+N}$	2	1	$+1$	$+1/2$
B_α	$\overline{M}$	$M+N$	1	2	-1	$+1/2$

TABLE I: The field content, in terms of chiral superfields, and its classical symmetries [15]. $SU(M) \times SU(M+N)$ is the gauge group ($M = kN$). An additional Z_2 symmetry exchanges $A \leftrightarrow B$ and conjugates the gauge fields.

- Superpotential (quasi-)marginal (exactly marginal when the two gauge groups are identical):

$$W = h\mathrm{Tr}\left[A_1B_1A_2B_2 - A_1B_2A_2B_1\right]$$

- Two couplings, beta-functions have opposite signs, Seiberg duality cascade from UV to IR:

$$SU(kN) \times SU((k+1)N) \to SU(kN) \times SU((k-1)N) \to SU((k-2)N) \times SU((k-1)N) \to \cdots$$

- Confinement (gaugino condensation) at dynamical scale set by scale anomaly (dim. transmutation).

I. R. Klebanov, M.J. Strassler, hep-th/0007191
M.J. Strassler, hep-th/0505153
A. Dimarsky, I.R. Klebanov, N. Seiberg, hep-th/0511254
J. Maldacena, D. Martelli, arXiv:0906.0591

The KS baryonic branch: Quiver QFT

- Perturbative treatment (illustrative purposes). Baryonic condensate from solving F-term and D-term equations: moduli space, set B=0 and solve

$$\mathrm{Tr}\left[T_q^A\left(A_1A_1^\dagger+A_2A_2^\dagger\right)\right]=(q+1)|c|^2\mathrm{Tr}\,T_q^A=0$$

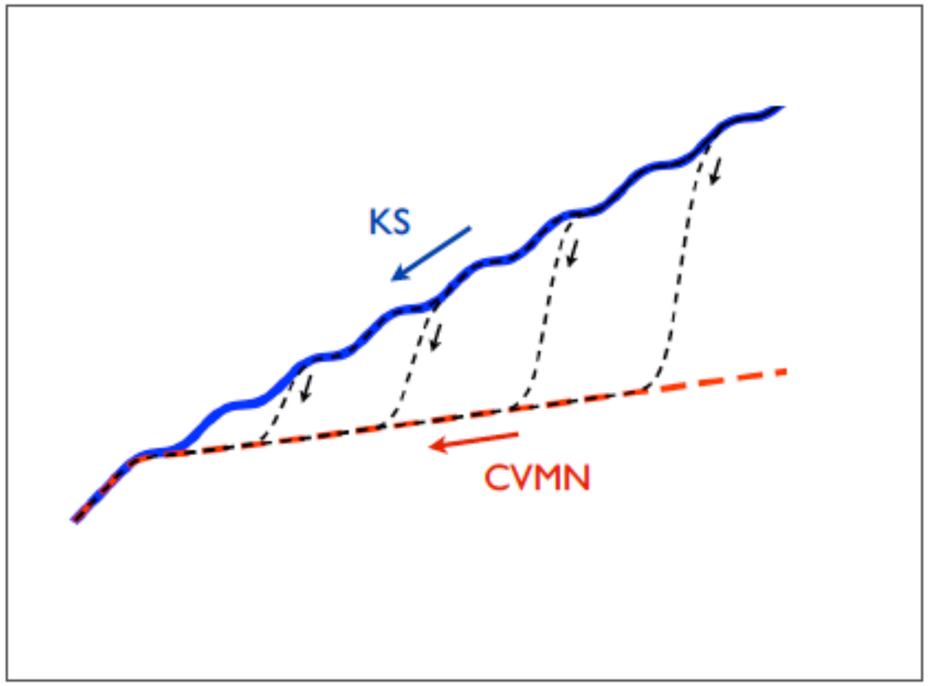
- solution can be written in terms of block-diagonal matrices:

$$\Phi_1 = \begin{pmatrix} \sqrt{q} & 0 & \cdots & 0 & 0 & 0 \\ 0 & \sqrt{q-1} & \cdots & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \sqrt{2} & 0 & 0 \\ 0 & 0 & \cdots & 0 & 1 & 0 \end{pmatrix}$$

$$\Phi_2 = \begin{pmatrix} 0 & 1 & \cdots & 0 & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 0 & \sqrt{q-1} & 0 \\ 0 & 0 & \cdots & 0 & 0 & \sqrt{q} \end{pmatrix}$$

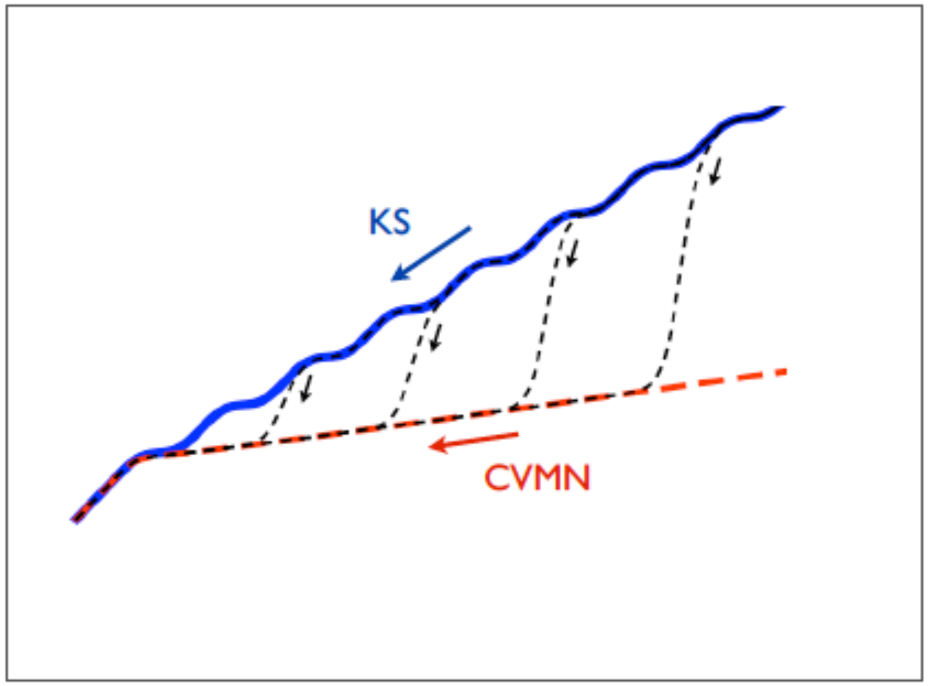
- Baryonic operator (baryonic branch): $\mathcal{U} = \frac{1}{q(q+1)N}\mathrm{Tr}\left[A_i^\dagger A_i - B_iB_i^\dagger\right]$
- Higgsing $SU(qN)\times SU((q+1)N)\to SU(N)$
- Perturbative spectrum (exercise): $M^2=g^2|c|^2\lambda_{\ell,\pm}$ $\quad \lambda_{\ell,\pm} = q+\frac{1}{2}\pm\sqrt{\left(q+\frac{1}{2}\right)^2-\ell(\ell+1)}$
- Spectrum organizes in irreps of SU(2), deconstruction of 2-sphere in internal space.

The KS baryonic branch: Quiver QFT



- 1-parameter family of solutions, the parameter is q.
- In far-UV all solutions undergo duality cascade.
- When group is reduced to SU(qN)xSU((q+1)N), cascade stops, baryonic condensate forms, higgsing.
- At lower energies, theory looks 6-dimensional: 4 Minkowski, plus deconstructed sphere.
- At very low energies, gaugino condensate forms, four-dimensional theory of zero-modes on sphere confines.
- Moduli space: q can be dialled freely, inequivalent solutions of same equations.

The KS baryonic branch: Quiver QFT



- Can we see evidence of deconstruction non-perturbatively?
- Multi-scale theory: two distinct condensates form, parametric separation (q).
- Scale invariance broken explicitly by scale anomaly, i.e. by gaugino condensate.
- Scale invariance broken spontaneously by baryonic condensate.
- Is there a dilaton in the limit in which the two are hierarchical?

The KS baryonic branch: supergravity

- Papadopoulos-Tseytlin ansatz: type-IIB system, internal 5-dimensional the base of the conifold T(1,1).
- Five-dimensional description in terms of 8 real scalars coupled to gravity.

$$\int d^5x\sqrt{-g_5}\left\{\frac{R}{4}-\frac{1}{2}G_{ab}g^{MN}\partial_M\Phi^a\partial_N\Phi^b - V(\Phi^a)\right\}$$

G.Papadopoulos, A.A. Tseytlin, hep-th/0012034

Berg, Haack, Mueck hep-th/0507285

$$\Phi^a = (\tilde{g}, p, x, \phi, a, b, h_1, h_2)$$

- Only two parameters: N, M (related to field theory quiver gauge groups). Action otherwise fixed.

$$\begin{aligned} G_{ab}\partial_M\Phi^a\partial_N\Phi^b = {} & \frac{1}{2}\partial_M\tilde{g}\partial_N\tilde{g} + \partial_M x\partial_N x + 6\partial_M p\partial_N p \\ & + \frac{1}{4}\partial_M\phi\partial_N\phi + \frac{1}{2}e^{-2\tilde{g}}\partial_M a\partial_N a + \frac{1}{2}N^2 e^{\phi-2x}\partial_M b\partial_N b \\ & + \frac{e^{-\phi-2x}}{e^{2\tilde{g}}+2a^2+e^{-2\tilde{g}}(1-a^2)^2}\left[\frac{1}{2}(e^{2\tilde{g}}+2a^2+e^{-2\tilde{g}}(1+a^2)^2)\partial_M h_2\partial_N h_2 \right. \\ & \left. +(1+2e^{-2\tilde{g}}a^2)\partial_M h_1\partial_N h_1 + 2a(e^{-2\tilde{g}}(a^2+1)+1)\partial_M h_1\partial_N h_2\right], \end{aligned}$$

$$\begin{aligned} V(\Phi^a) = {} & -\frac{1}{2}e^{2p-2x}(e^{\tilde{g}}+(1+a^2)e^{-g}) + \frac{1}{8}e^{-4p-4x}(e^{2\tilde{g}}+(a^2-1)^2e^{-2\tilde{g}}+2a^2) \\ & + \frac{1}{4}a^2e^{-2\tilde{g}+8p} + \frac{1}{8}N^2e^{\phi-2x+8p}\left[e^{2\tilde{g}}+e^{-2\tilde{g}}(a^2-2ab+1)^2+2(a-b)^2\right] \\ & + \frac{1}{4}e^{-\phi-2x+8p}h_2^2 + \frac{1}{8}e^{8p-4x}(M+2N(h_1+bh_2))^2\,. \end{aligned}$$

- Full lift to 10 dimensional type IIB supergravity known explicitly (gravity, F(3), F(5), B(2) ...)

The KS baryonic branch: supergravity

A. Ceresole, G. Dall'Agata, R. D'Auria, S. Ferrara, hep-th/9905226
F. Bigami, L. Girardello, A. Zaffaroni, hep-th/0011041
F. Bigazzi, A. Cotrone, M. Petrini, A. Zaffaroni, hep-th/0303191
D. BAumann, A. Dymarsky, S. Kachru, I.R. Klebanov, arXiv:1001.5028

- Scalars classified in term of operators in dual gauge theory

Φ^a	Δ	$\mathcal{O}$ [29]	BPS	KS	$k_2 = 0$	$0 < k_2 < e^{-\Phi_\infty}$	$k_2 = e^{-\Phi_\infty}$	Butti et al. [7]
a	1,3	$\mathrm{Tr}\,(W_1^2 - W_2^2)$	3	3	3	3	3	3
$\tilde{g}$	2	$\mathrm{Tr}\,(A\bar{A} - B\bar{B})$	2				2	2
Φ	0,4	$\mathrm{Tr}\,(F_1^2 + F_2^2)$	0	0	0	0	0	0
h_1	0,4	$\mathrm{Tr}\,(F_1^2 - F_2^2)$	0			0		
$x,\ p$	-4,-2,6,8	$\mathrm{Tr}\,W^2\bar{W}^2$	-4,6		-4,6	-4,6	6	
$b,\ h_2$	1,3	$\mathrm{Tr}\,(W_1^2 + W_2^2)$	3					
	-3,7	$\mathrm{Tr}\,(A\bar{A} + B\bar{B})W^2$	-3					

TABLE I: Field-theory operator analysis, based on expanding in the UV near the KW fixed points. The columns show the five-dimensional fields, the scaling dimensions of the perturbations they allow, the corresponding field-theory operators in terms of the two gauge groups, the scaling dimensions selected by the BPS equations. The last five columns show which couplings or VEVs correspond to the *independent* integration constants that can be dialed, labelled by the corresponding scaling-dimension of the gravity-dual scalar. Notice that some of the couplings/VEVs that are not explicitly highlighted are present, but their UV-boundary values are not independent, in particular the dimension-2 VEV is present in all the solutions constructed starting from the master equation.

- Many solutions known: Klebanov-Witten (CFT), Klebanov-Tseytlin (cascade), Klebanov-Strassler (cascade+confinement), Maldacena-Nunez (twisted 2-sphere, confinement), …

- Our main interest: baryonic branch of KS, 1-parameter family of solution, both dimension-2 and dimension-3 operator condense, 10-dimensional metric completely smooth in the IR, end of space (area-law confinement), well-behaved, close to being asymptotically AdS in UV (cascade).

A. Butti, M. Grana, R. Minasian, M. Petrini, A. Zaffaroni, hep-th/0412187

- Spectrum of fluctuations never computed before: serious technical challenge, background known only numerically, nasty coupled equations, presence of important subdominant (exponentially suppressed) contributions, numerical fine-tuning.

The KS baryonic branch: supergravity

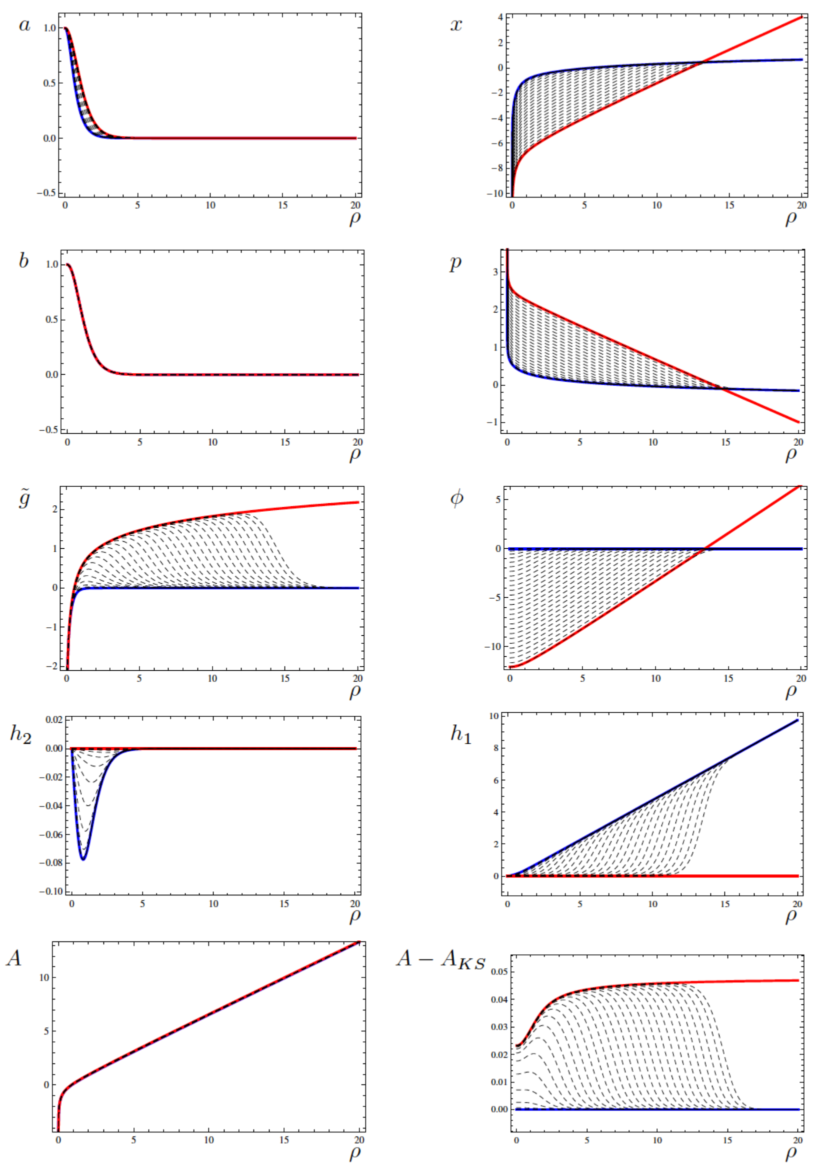


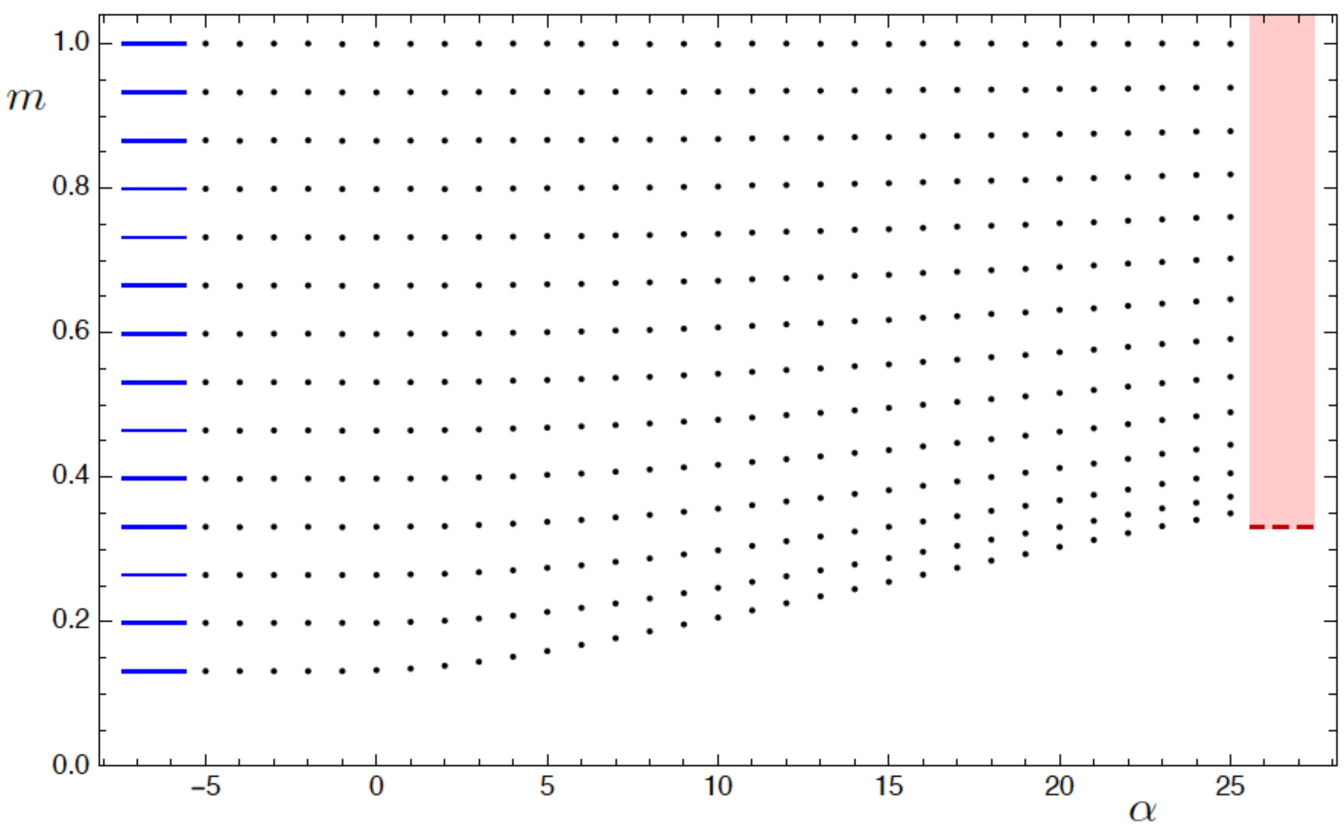
Figure 2. All the background functions for several examples of backgrounds: in blue we exhibit the KS solution, in black (dashing) several examples of baryonic branch solutions with different values of κ_1 (and hence $\bar{\rho}$), and in red the CVMN solution. See the main text for clarifications about the choices of integration constants adopted. Notice that in the bottom left panel all curves for A are on top of each other. To make visible the small differences, we show in the bottom-right panel the difference $A - A_{KS}$, on a much smaller scale.

- Known solution-generating technique: BPS equations can be reduced to one second-order ODE, the solution of which can be used algebraically to generate all the background functions.
- One parameter family of solutions identified by matching UV and IR expansions to numerical solutions.
- Checked that at the extrema of the baryonic branch we could reproduce CVMN and KS solutions, respectively.
- Important mixing effects, IR behaviour non-trivial, several background functions diverge. (but no singularity in 10 dimensions).

The KS baryonic branch: spectrum of glueballs

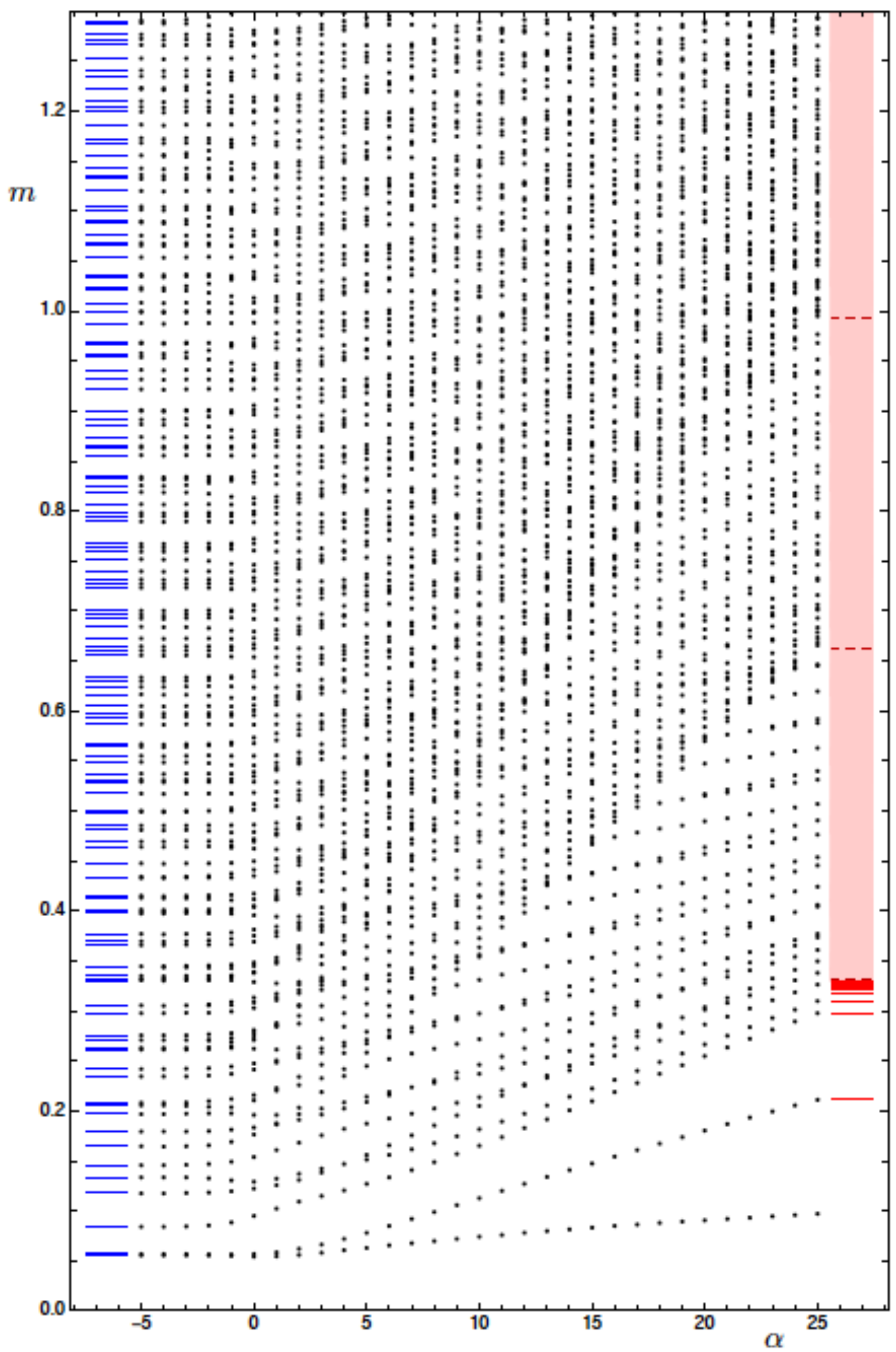
- Fluctuations of the solutions rewritten in terms of gauge-invariant variables.
- IR and UV asymptotic solutions used in combination with boundary conditions (analogy with improvement on the lattice).
- Extensive study of IR-boundary (cutoff) effects (analogy with finite size in lattice calculations)
- Extensive study of UV-boundary (cutoff) effects (analogy with continuum-limit extrapolation in lattice calculations)
- Mid-determinant method implemented to find solutions.
- Final results completely independent of cutoff effects: boundaries removed, spectrum due to bulk geometry only.
- Checked that (known) KS and CVMN results recovered (plus additional towers).
- All technical details is Appendix of paper (15 pages of it…)

The KS baryonic branch: spectrum of glueballs



- Tensor spectrum as function of position along baryonic branch.
- Blue: KS spectrum. Agreement
- Pink: CVMN spectrum. Agreement asymptotically with continuum spectrum.
- Large m: KS cascade recovered, typical supergravity spectrum, unaffected.
- Lower m: lightest states moved up and squeezed together.
- Evidence in non-perturbative spectrum of intermediate regime in which spectrum of mass anomalously dense: deconstruction.
- Big mass gap, no light states. As expected.

The KS baryonic branch: spectrum of glueballs



- Scalar spectrum as function of position along baryonic branch.
- Blue: KS spectrum. Agreement
- Pink: CVMN spectrum. Agreement asymptotically with mixed discrete/continuum spectrum, with three separate thresholds.
- Large m: KS cascade recovered, typical supergravity spectrum, unaffected.
- Lower m: lightest states moved up and squeezed together.
- Evidence in non-perturbative spectrum of intermediate regime in which spectrum of mass anomalously dense: deconstruction.
- One state separates out: light scalar particle (most likely dilaton).

The KS baryonic branch: spectrum of glueballs

- Beautiful realisation of dimensional deconstruction: on gravity side, we performed 5-dimensional calculation with 8 scalars coupled to gravity, which corresponds in 4-dimensional field theory language to spectrum of glueballs of quiver KS theory along baryonic branch, spectrum shows expected four-dimensional behaviour of eigenvalues for small m and large m, but at intermediate m becomes very dense. Deconstruction of two-sphere, finite-size representation of SU(2).

- Beautiful realisation of multi-scale dynamical system with light scalar composite particle. The field theory is known, and well understood, the supergravity is known, and well understood. There are no singularities, asymptotic behaviour of the backgrounds are healthy. Calculation of the spectrum can be done (expensive, possibly…), yields light scalar.

- Interpretation of light scalar as dilation on the basis of field theory argument only, for the time being.

Outlook

- Numerics: can we go closer to the CVMN system, and study better the behaviour of the theory?

- Spectrum: can we extend the consistent truncation, including also vector and pseudo-scalar modes? In particular, might be important to understand higgsing process. D. Cassani and A. Faedo, arXiv:1008.0883
I. Bena, G. Giecold, M. Grana, N. Halmagyi, F. Orsi, arXiv:1008.0983

- What is the role of supersymmetry? Purely technical, or is there a connection between light scalar and higgs phenomenon?

- What is the role of U(1)-baryon breaking? Where is the axion? S.S. Gubser, C.P. Herzog, I. Klebanov hep-th/0405282

- Can we compute couplings, to prove that the light scalar is a dilaton?

- Phenomenology: can one implement electroweak symmetry breaking? Probe D-branes

- Are there other cases of this type? To the best of our knowledge, this is the only known calculable example of a well-defined and well-understood four-dimensional QFT that yields a parametrically light pseudo-dilaton.

- Indications of similar behaviour started to emerge on the lattice: SU(3) with Nf=8, or with 2 sextets.

Z. Fodor et al. arXiv:1209.0391, 1502.00028, 1605.08750
T. Appelquist et al. arXiv:1601.04027

T. Appelqusit, J. Ingoldby, MP, arXiv:1702.04410, 1711.00067