Analysis of the spectrum of composite resonances

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based on arXiV: 1610.09293 [Phys.Rev. D95 (2017)]
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Montpellier workshop on strong dynamics at the EW scale
7 December 2017

Underlying composite Higgs dynamics

Barring extra space-time dimensions

$\Rightarrow$ Simplest, well-understood, explicit realization provided by gauge theory of fermions that confines at the multi-TeV scale Λ

Matter content

EW sector (Higgs as pNGB) + coloured sector (top partners)

Minimal model [Barnard et al, '13]

- EW sector: 4 Weyl fermions ψ in pseudoreal irreps of hypercolour
  $\Rightarrow$ $SU(4)/Sp(4)$ pattern of symmetry breaking ($\psi \sim \square_{Sp(2N)}$)
- Coloured sector: 6 Weyl fermions X in real irreps
  $\Rightarrow$ $SU(6)/SO(6)$ ($X \sim \begin{smallmatrix}\square\\\square\end{smallmatrix}_{Sp(2N)}$)

Explicit breaking sources

Need to destabilise Higgs potential to break EW symmetry:

- Gauging $\rightarrow$ can not destabilises Higgs potential
- Partial compositeness
- Current masses: $m_{\psi,X}$

Full gauge theory (hypergluons, hyperfermions as d.o.f) hard to study below Λ because of its non-perturbative nature $\Rightarrow$ Effective models are useful

- Chiral Lagrangians: dictated only by global symmetries

$$\mathcal{L}_{\chi PT} = \frac{F_G^2}{4}\langle (D_\mu U)^\dagger D^\mu U \rangle \qquad U = \exp(2iG^{\hat{A}}T^{\hat{A}}/F_G)\Sigma_\epsilon$$

$\Rightarrow$ Little information on the details of the strong dynamics

$\Rightarrow$ Not sure that an UV completion exists [e.g. $SO(5)/SO(4)$]

- 4-fermion interactions (gauge bosons froze-out)

$$\mathcal{L}_{NJL} = (\overline{\Psi}\Gamma^\alpha\Psi)(\overline{\Psi}\Gamma_\alpha\Psi)$$

[Nambu and Jona-Lasinio '61]

$\Rightarrow$ Definite UV completion and underlying gauge symmetry respected

$\Rightarrow$ Possible to compute non-perturbative quantities (like LECs) with NJL techniques

$\Rightarrow$ Estimation of the composite resonances masses (mesons and baryons)

Λ

gauge theory

(fundamental fermions, techni-gluons)

$$\mathcal{L}_{\rm gauge} = \overline{\Psi} i \not{D} \Psi - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}$$

Exact results on the strong dynamics exist

Intermediate scale effective models

(fermionic bilinears)

$$\mathcal{L}_{\rm NJL} = \overline{\Psi} i \not{\partial} \Psi + \sum_\alpha \left(\overline{\Psi}\Gamma^\alpha \Psi\right)\left(\overline{\Psi}\Gamma_\alpha \Psi\right)$$

NJL techniques (Large N), ...

f

Low scale effective models

(Goldstone bosons, ...)

$$\mathcal{L}_{\chi PT} = \frac{f^2}{2} \partial_\mu \Pi \partial^\mu \Pi + \cdots$$

Chiral perturbation theory

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NJL approx of strong dynamics: 'froze out' hypergluons induce 4-fermion interactions

Scalar 4-fermion operators

Relevant for the spontaneous breaking and spin 0 mesons masses:

$$\mathcal{L}^{\psi}_{scal} = \frac{\kappa_A}{2N}(\psi^a\psi^b)(\overline{\psi}_a\,\overline{\psi}_b) + \frac{\kappa_B}{8N}\left[\epsilon_{abcd}(\psi^a\psi^b)(\psi^c\psi^d) + h.c.\right]$$

- $\kappa_{A,B} \sim 1/\Lambda^2$ real, dimensionful couplings
- κ_A controls spontaneous symmetry breaking $SU(4) \to Sp(4)$
- κ_B explicitly breaks the anomalous $U(1)_\psi$ symmetry

Vector and axial-vector 4-fermion operators

$$\mathcal{L}^{\psi}_{vect} = \frac{\kappa_C}{2N}\left(\overline{\psi}\,T^0_\psi\,\overline{\sigma}^\mu\psi\right)^2 + \frac{\kappa_D}{2N}\left(\overline{\psi}T^A\overline{\sigma}^\mu\psi\right)^2 + \frac{\kappa_D}{2N}\left(\overline{\psi}\,T^{\hat{A}}\overline{\sigma}^\mu\psi\right)^2$$

$\Rightarrow$ Non-tachyonic masses for $\kappa_{C,D} > 0$ (consistent with current-current hypothesis)

$\Rightarrow$ Additional spin 1 resonances associated to $(\psi^a\sigma^{\mu\nu}\psi^b) \sim 10_{Sp(4)}$ do not appear at the level of four-fermion interactions

		Colour	Flavour	
	Lorentz	$Sp(2N)$	$SU(4)$	$Sp(4)$
Hypercolour fermions ψ_i^a	$(1/2,0)$	$\square_i$	4^a	4
$\overline{\psi}_{ai} \equiv \psi^\dagger_{aj}\Omega_{ji}$	$(0,1/2)$	$\square_i$	$\overline{4}_a$	4^*
Spin-zero bilinears $M^{ab} \sim (\psi^a\psi^b)$	$(0,0)$	1	6^{ab}	$5+1$
$\overline{M}_{ab} \sim (\overline{\psi}_a\overline{\psi}_b)$	$(0,0)$	1	$\overline{6}_{ab}$	$5+1$
Spin-one bilinears $a^\mu \sim (\overline{\psi}_a\overline{\sigma}^\mu\psi^a)$	$(1/2,1/2)$	1	1	1
$(V^\mu, A^\mu)^b_a \sim (\overline{\psi}_a\overline{\sigma}^\mu\psi^b)$	$(1/2,1/2)$	1	15^a_b	$10+5$

Hypercolour-invariant fermionic bilinears have the quantum numbers of the meson resonances

Lightest composite meson resonances

Scalars: $\sigma + S^{\hat{A}} \sim 1+5$

Pseudo-scalars: $\eta' + G^{\hat{A}} \sim 1+5$

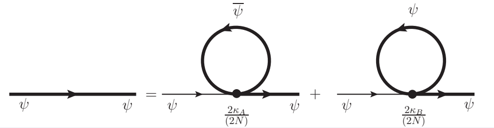
Vectors: $V^A_\mu \sim 10$

Axial-vector: $a_\mu + A^{\hat{A}}_\mu \sim 1+5$

Lagrangian can be rewritten in the 'physical' channels, corresponding to definite $Sp(4)$ representations using $SU(4)$ Fierz identities:

$$\begin{aligned}\mathcal{L}^{\psi}_{scal} &= 2\frac{\kappa_A}{(2N)}\left[\left(\psi\Sigma_0 T^0_\psi\psi\right)\left(\overline{\psi}T^0_\psi\Sigma_0\overline{\psi}\right)+\left(\psi\Sigma_0 T^{\hat{A}}\psi\right)\left(\overline{\psi}T^{\hat{A}}\Sigma_0\overline{\psi}\right)\right]\\ &+ \frac{\kappa_B}{(2N)}\left[\left(\psi\Sigma_0 T^0_\psi\psi\right)\left(\psi\Sigma_0 T^0_\psi\psi\right)-\left(\psi\Sigma_0 T^{\hat{A}}\psi\right)\left(\psi\Sigma_0 T^{\hat{A}}\psi\right)+h.c.\right]\end{aligned}$$

Schwinger Dyson equation determines dynamical fermion mass M_ψ



$$M_\psi = 4(\kappa_A+\kappa_B)M_\psi\tilde{A}_0(M_\psi^2)$$

Self-consistence implicitly ressums all diagrams leading in 1/N

$$\xi \equiv \frac{\Lambda^2(\kappa_A+\kappa_B)}{4\pi^2} = \left[1-\frac{M_\psi^2}{\Lambda^2}\ln\left(\frac{\Lambda^2+M_\psi^2}{M_\psi^2}\right)\right]^{-1}$$

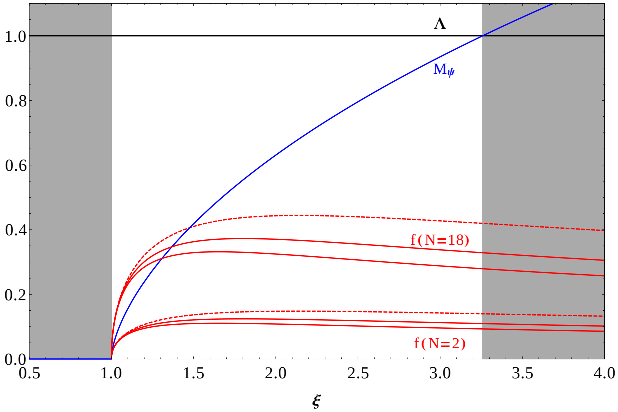
critical coupling $\quad 1<\xi\lesssim 3.25 \quad$ maximal coupling

- Non trivial solution $M_\psi \neq 0$ ($SU(4)$ spontaneously broken) exists only if $\xi > 1$
- Consistent resummation: $0 < M_\psi/\Lambda \lesssim 1$

$$\langle vac|\mathcal{J}_\mu^{\hat{A}}(0)|G^{\hat{B}}(p)\rangle = ip_\mu \frac{f}{\sqrt{2}}\delta^{\hat{A}\hat{B}}$$

EW precision observables receive order v^2/f^2 corrections $\Rightarrow f \gtrsim 0.5 - 1$ TeV

$$\frac{f^2}{2} = \lim_{q^2\to 0}[-q^2\overline{\Pi}_A(q^2)] = \frac{\tilde{\Pi}_A(0)}{1 + 2\kappa_D\tilde{\Pi}_A(0)/N}, \qquad \tilde{\Pi}_A(0) = -2(2N)M_\psi^2\tilde{B}_0(0, M_\psi^2)$$

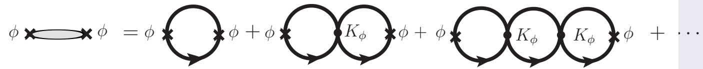


- f residue of the Goldstone boson pole in the resummed transverse axial correlator
- f sets the scale of the composite sector
- $f \propto \sqrt{N}$
- f can be as small as $\Lambda/10$ ($\Lambda \equiv$ NJL cutoff)

$\Rightarrow$ possibly large hierarchy

Bethe-Salpether equation

Resummation (geometrical series) of an infinite number of constituent fermion loops at leading order in $1/N$ $\Rightarrow$ Two-point correlators develop a pole



The pole defines the meson mass M_ϕ

$$\overline{\Pi}_\phi(q^2) = \frac{\tilde{\Pi}_\phi(q^2)}{1 - 2K_\phi \tilde{\Pi}_\phi(q^2)} \longrightarrow 1 - 2K_\phi \tilde{\Pi}_\phi(q^2 = M_\phi^2) = 0$$

ϕ	K_ϕ	$\tilde{\Pi}_\phi(q^2)$
$G^{\hat{A}}$	$2(\kappa_A + \kappa_B)/(2N)$	$\tilde{\Pi}_P(q^2) = (2N)\left[\tilde{A}_0(M_\psi^2) - \frac{q^2}{2}\tilde{B}_0(q^2, M_\psi^2)\right]$
η'	$2(\kappa_A - \kappa_B)/(2N)$	
$S^{\hat{A}}$	$2(\kappa_A - \kappa_B)/(2N)$	$\tilde{\Pi}_S(q^2) = (2N)\left[\tilde{A}_0(M_\psi^2) - \frac{1}{2}(q^2 - 4M_\psi^2)\tilde{B}_0(q^2, M_\psi^2)\right]$
σ	$2(\kappa_A + \kappa_B)/(2N)$	

and similarly for the spin one channels V and A

No confinement in the NJL ⇒ Prescription for the unphysical imaginary parts

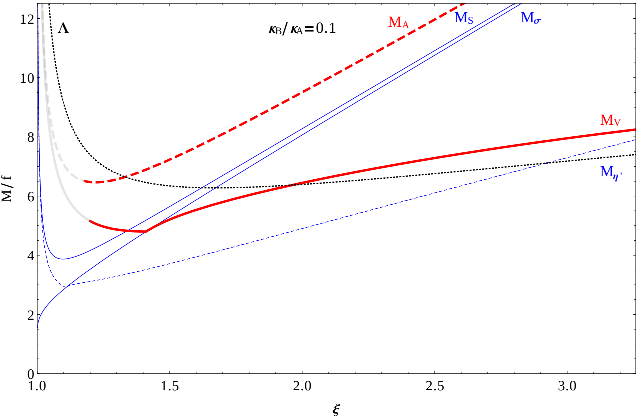
$$1 - 2K_\phi \tilde{\Pi}_\phi(q^2) = c_0^\phi(q^2) + c_1^\phi(q^2) q^2 \quad \longrightarrow \quad M_\phi^2 = Re\left[-\frac{c_0^\phi(M_\phi^2)}{c_1^\phi(M_\phi^2)}\right]$$

$K_\phi \equiv$ four-fermion couplings $\quad \tilde{\Pi}_\phi(q^2) \equiv$ Polarisation amplitudes

- Inserting the gap-equation, one recovers consistently the Goldstone pole: $M_G = 0$
- Singlet pseudo-scalar proportional to $U(1)$ anomaly and mixes with axial vector: $$M_{\eta'}^2 = -\frac{\kappa_B}{\kappa_A^2 - \kappa_B^2} \frac{[1 - 2K_a \tilde{\Pi}_A^L(M_{\eta'}^2)]}{\tilde{B}_0(M_{\eta'}^2, M_\psi^2)}$$
- Scalars proportional to the mass gap M_ψ: $$M_\sigma^2 = 4M_\psi^2, \quad M_S^2 = 4M_\psi^2 + M_{\eta'}^2 \frac{\tilde{B}_0(M_{\eta'}^2, M_\psi^2)}{\tilde{B}_0(M_S^2, M_\psi^2)} \simeq M_\sigma^2 + M_{\eta'}^2$$
- Vector heavy even for vanishing mass gap: $$M_V^2 = \frac{-3}{4\kappa_D \tilde{B}_0(M_V^2, M_\psi^2)} + 2M_\psi^2 \frac{\tilde{B}_0(0, M_\psi^2)}{\tilde{B}_0(M_V^2, M_\psi^2)} - 2M_\psi^2$$
- Axial-vector generally the heaviest: $$M_A^2 = \frac{-3}{4\kappa_D \tilde{B}_0(M_A^2, M_\psi^2)} + 2M_\psi^2 \frac{\tilde{B}_0(0, M_\psi^2)}{\tilde{B}_0(M_V A2, M_\psi^2)} + 4M_\psi^2 \simeq M_V^2 + 6M_\psi^2$$

Current-current hypothesis

- Large-N relation among 4-fermion operators dominated by single hypergluon exchange $\to \kappa_A = \kappa_C = \kappa_D$ $\qquad (M_a = M_A)$



- $M_\phi/f \sim 1/\sqrt{N}$ ($N = 4$ here)
- Free parameters: $\xi = \Lambda^2(\kappa_A + \kappa_B)/(4\pi^2)$, κ_B/κ_A
- EW splitting neglected (e.g. $\mathbf{5}_{Sp(4)} = \mathbf{2}_{\pm 1/2}\mathbf{1}_0$) $\Rightarrow$ Full $Sp(4)$ multiplets
- Consistently recover NGBs: $M_G = 0$

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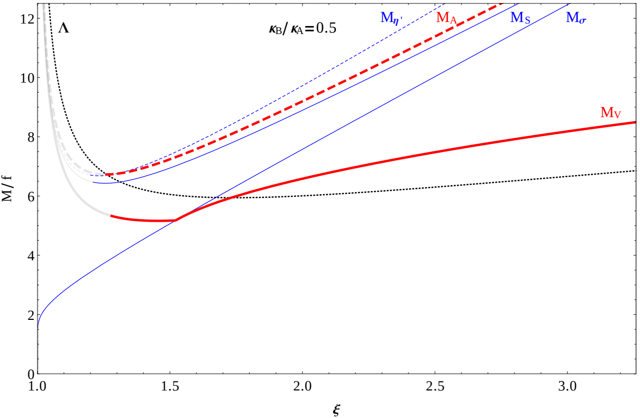

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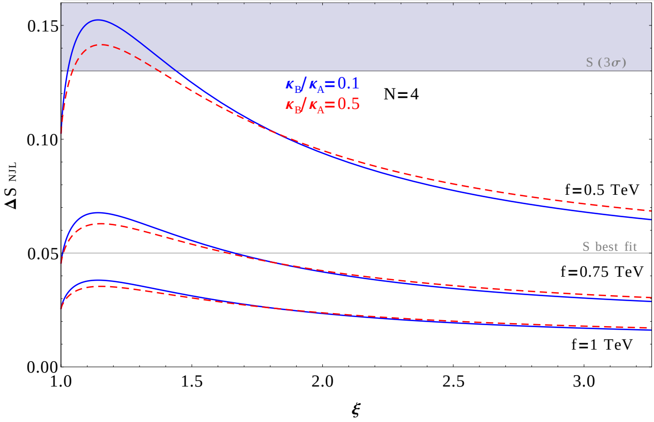
NJL estimation of S parameter

S parameter

Need only to assume vev for the Higgs (No need to explicitly consider details of breaking terms)

$$\Delta S = 16\pi \left. \frac{d\Pi^{(v)}_{3Y}(q^2)}{dq^2} \right|_{q^2=0} = 8\pi \frac{v^2}{f^2} \frac{d}{dq^2} \left. \left(q^2 \Pi_{V-A}(q^2) \right) \right|_{q^2=0}, \; \frac{v}{f} = \sin\left(\frac{\langle h \rangle}{f} \right)$$

Correlator $\Pi_{V-A}(q^2)$ can be estimated in the NJL approximation



3σ limit assumes $\Delta T = 0$

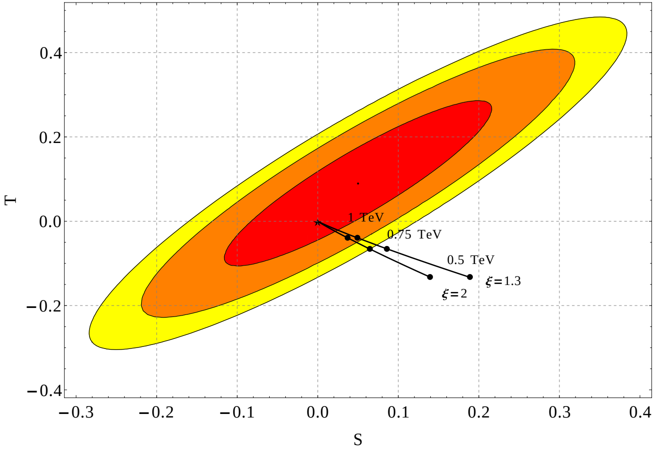
ΔS decreases when strong sector decouples (ie increase of f)

No corresponding shift in T parameter due to custodial symmetry

IR contributions

Composite sector also modifies Higgs couplings to EW gauge bosons by factor $\sqrt{1 - v^2/f^2}$

$$\Delta S_{\mathrm{IR}} = \frac{1}{6\pi}\frac{v^2}{f^2}\ln\left(\frac{\mu}{M_h}\right), \qquad \Delta T_{\mathrm{IR}} = -\frac{3}{8\pi}\frac{1}{\cos^2\theta_W}\frac{v^2}{f^2}\ln\left(\frac{\mu}{M_h}\right) = -\frac{9}{4}\frac{\Delta S_{\mathrm{IR}}}{\cos^2\theta_W}$$



One expects additional contributions from partial compositeness
⇒ Not complete prediction, only shown is specific contributions

Adding the coloured sector

Introduce new constituent coloured fermions X^f to form spin-1/2 baryons mixing with SM top quark

$\Rightarrow$ Need to go beyond $Sp(2N)$ fundamental representation: $X^f \sim \genfrac{}{}{0pt}{}{\square}{\square}$ $(N \geqslant 2)$

Minimal cases:
- $Sp(2) \cong SU(2)$ (EW sector alone) $\rightarrow$ lattice results available
- $Sp(4) \cong SO(5)$ (EW+ coloured sectors)

	Lorentz	$Sp(2N)$	$SU(6)$	$SO(6)$
X^f_{ij}	$(1/2,0)$	$\genfrac{}{}{0pt}{}{\square}{\square}_{ij}$	6^f	6
$\overline{X}_{fij} \equiv \Omega_{ik} X^\dagger_{fkl} \Omega_{lj}$	$(0,1/2)$	$\genfrac{}{}{0pt}{}{\square}{\square}_{ij}$	$\overline{6}_f$	6
$M^{fg}_c \sim (X^f X^g)$	$(0,0)$	1	21^{fg}	$20'+1$
$\overline{M}_{cfg} \sim (\overline{X}_f \overline{X}_g)$	$(0,0)$	1	$\overline{21}_{fg}$	$20'+1$
$a^\mu_X \sim (\overline{X}^f \bar\sigma^\mu X_f)$	$(1/2,1/2)$	1	1	1
$(V^\mu_c, A^\mu_c)^g_f \sim (\overline{X}_f \bar\sigma^\mu X^g)$	$(1/2,1/2)$	1	35^f_g	$15+20'$

Lightest coloured resonances

Scalars: $\sigma_X + S^{\hat{F}}_c \sim 1 + 20'$

Pseudo-scalars: $\eta_X + G^{\hat{F}}_c \sim 1 + 20'$

Vectors: $V^{\mu F}_c \sim 15$

Axial-vector: $a^\mu_c + A^{\mu \hat{F}}_c \sim 1 + 20'$

$$20'_{SO(6)} = (8 + 6 + \overline{6})_{SU(3)_c} \qquad 15_{SO(6)} = (1 + 8 + 3 + \overline{3})_{SU(3)_c}$$

$U(1)$ (anomalous) symmetries

Lot of changes appears when theory includes both EW and coloured sectors

- Important to consider global fermion numbers $U(1)_\psi$ and $U(1)_X$
- Currents $\mathcal{J}^0_{\mu\psi,X}$ both anomalous w.r.t $Sp(2N)$ (like $U(1)_A$ in QCD)
- However, one linear combination is anomaly free and thus conserved:

$$\mathcal{J}^0_\mu = \mathcal{J}^0_{\mu X} - 3(N-1)\mathcal{J}^0_{\mu\psi}$$

$\Rightarrow$ New Goldstone boson η_0 appears while η' receive a mass from the anomaly

Construct the minimal operator that preserves all exact symmetries but explicitly breaks the anomalous $U(1)$ (generalisation of κ_B-term)

- EW sector: $Sp(2N)$ anomaly breaks $U(1)_\psi \to \mathcal{O}_\psi = -\frac{1}{4}\epsilon_{abcd}(\psi^a\psi^b)(\psi^c\psi^d)$
- Colour sector: anomaly breaks $U(1)_X \to \mathcal{O}_X = -\frac{1}{6!}\epsilon_{f_1\cdots f_6}\epsilon_{g_1\cdots g_6}(X^{f_1}X^{g_1})\cdots(X^{f_6}X^{g_6})$
- Full theory preserves $U(1)_{X-3(N-1)\psi}$: $\to \mathcal{L}_{\psi X} = A_{\psi X}\frac{\mathcal{O}_\psi}{(2N)^2}\left[\frac{\mathcal{O}_X}{[(2N+1)(N-1)]^6}\right]^{(N-1)}$

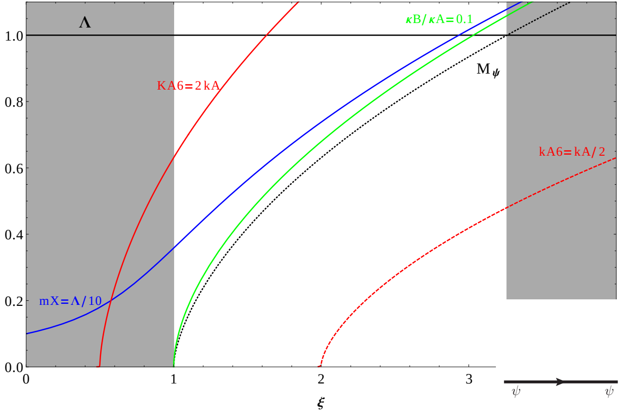
After spontaneous breaking $\mathcal{L}_{\psi X}$ generates effective 4-fermion operators ψ^4, X^4 and ψ^2X^2

Two coupled mass gap equations:

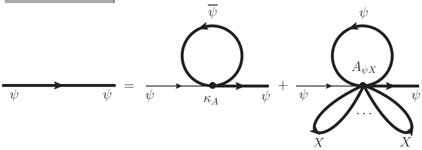
$$\begin{cases} M_\psi = 4\left[\kappa_A + \kappa_B(M_X^2)\right] M_\psi \tilde{A}_0(M_\psi^2) \\ M_X = 4\left[\kappa_{A6} + \kappa_{B6}(M_\psi^2, M_X^2)\right] M_X \tilde{A}_0(M_X^2) + m_X \end{cases}$$

$$\begin{cases} \kappa_B = \kappa_{B6} = 0 \\ \kappa_A = \kappa_{A6}, m_X = 0 \end{cases}$$

$$\Rightarrow M_\psi = M_X$$

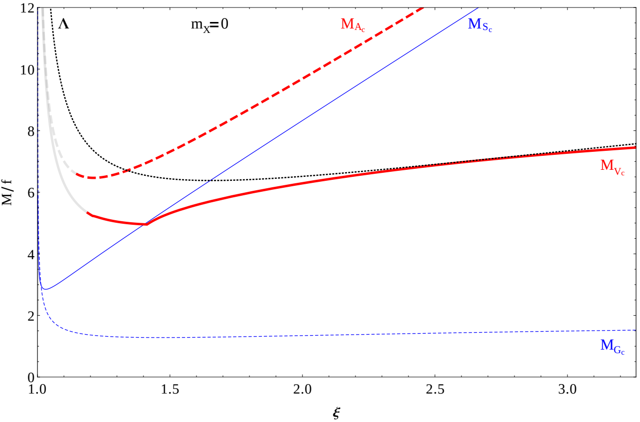


- Coloured sector window [between critical coupling ($M_X = 0$) and maximal coupling ($M_X = \Lambda$)] shifts respect to the EW sector window
- $m_X \neq 0$: No critical coupling as $M_X \geqslant m_X$



Current-current hypothesis

The ratio EW masses/ coloured masses strongly depends on the ratio κ_{A6}/κ_A
Unfortunately the large-N approximation does not determine this ratio uniquely (but still determines $\kappa_{A6} = \kappa_{C6} = \kappa_{D6}$)
$\Rightarrow$ Choose $\kappa_A = \kappa_{A6}$



- $M_\phi/f \sim 1/\sqrt{N}$ ($N = 4$, $\kappa_B/\kappa_A = 1/100$ here)
- Goldstone bosons receive a mass from gluon loops that evade bounds for $f \gtrsim 1$ TeV
- Goldstone (and coloured resonances) mass increase with f

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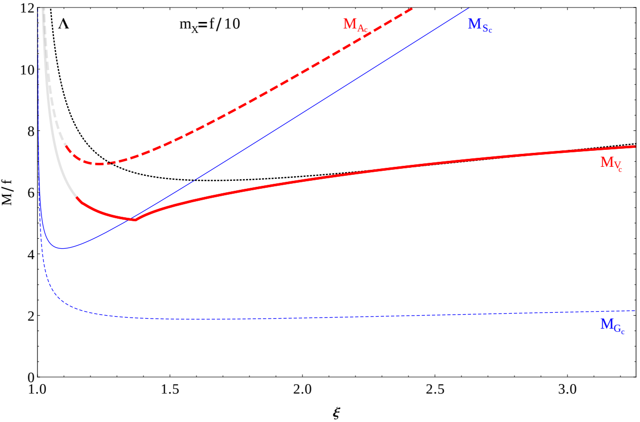

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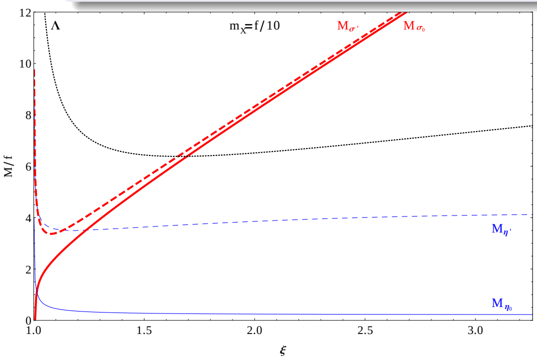
(Pseudo-)scalars: Anomalous operator $A_{\psi X}$ induces a coupling $\psi^2 X^2$ of the same order as the couplings ψ^4, X^4
$\Rightarrow$ One linear combination of η_0 is a pNGB (massless for $m_X = 0$)

$$\mathbf{K}_{\eta_\psi \eta_X} = \begin{pmatrix} K_{\eta_\psi} & -K_{\psi X} & 0 & 0 \\ -K_{\psi X} & K_{\eta X} & 0 & 0 \\ 0 & 0 & K_a & 0 \\ 0 & 0 & 0 & K_{a_c} \end{pmatrix}, \quad \mathbf{\Pi}_{\eta_\psi \eta_X} = \begin{pmatrix} \tilde{\Pi}_P^\psi & 0 & \sqrt{p^2}\tilde{\Pi}_{AP}^\psi & 0 \\ 0 & \tilde{\Pi}_P^X & 0 & \sqrt{p^2}\tilde{\Pi}_{AP}^X \\ \sqrt{p^2}\tilde{\Pi}_{AP}^\psi & 0 & \tilde{\Pi}_A^{L\psi} & 0 \\ 0 & \sqrt{p^2}\tilde{\Pi}_{AP}^X & 0 & \tilde{\Pi}_A^{LX} \end{pmatrix}$$



- pNGB η_0 could be very light
$\Rightarrow M_{\eta_0}^2 \sim m_X$

- Anomalous pseudoscalar η' could also be very light
$\Rightarrow M_{\eta_0}^2 \sim A_{\psi X}$
(No way to estimate $A_{\psi X}$)

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Calculation of top partners masses within NJL framework [work in progress]

⇒ Possibility to have light top partners for PC?

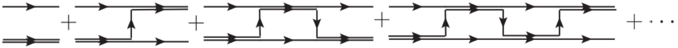
⇒ Relevant for more phenomenological approaches where mixing with top partners is included, e.g. if $t' \sim (5+1)_{Sp(4)}$

$$M_{top} = \begin{pmatrix} 0 & -y_{5L} f \cos^2\frac{\theta}{2} & y_{5L} f \sin^2\frac{\theta}{2} & \frac{y_{1L}}{\sqrt{2}} f s_\theta & 0 \\ \frac{y_{5R}}{\sqrt{2}} f s_\theta & M_5 & 0 & 0 & 0 \\ \frac{y_{5R}}{\sqrt{2}} f s_\theta & 0 & M_5 & 0 & 0 \\ -y_{1R} f c_\theta & 0 & 0 & M_1 & 0 \\ 0 & 0 & 0 & 0 & M_5 \end{pmatrix}$$

- NJL allows to estimate VL masses M_1 and M_5 and similarly for other embedding or models
  ⇒ What is the more intersting top partner? the most favorable one?

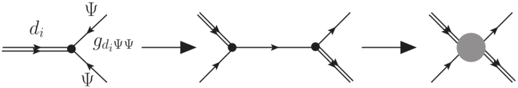
- Allows to discriminate different scenarios: gives an idea if $M_1 \simeq M_5$, $M_1 > M_5$ or $M_1 < M_5$

- Identify baryons $(\psi\psi X)$, $\cdots$ $\Rightarrow$ Lorentz, hypercolour and flavour contractions
- Approximate trilinear baryons as diquark-quark system:



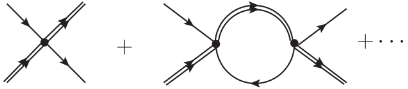
$\Rightarrow$ Compute diquarks masses with same techniques employed for mesons

- Static approximation: Neglect kinetic term of the exchange fermions



$\Rightarrow$ pinch diagrams to get fermi interactions between 2 diquarks and 2 fermions
$\Rightarrow$ Couplings of diquarks should also be extracted from the NJL ressumation

- Resum the geometrical series with loops of constituent fermions and diquark:



$\Rightarrow$ Loops involve two masses
$\Rightarrow$ Only diquarks bound states $(M_d < 2M_f)$ contribute to baryon mass

Summary

Thorough analysis of the spectrum of meson (and baryons) resonances in a confining gauge theory with fermions in two different hypercolour representations

- NJL well describes SSB: non-perturbative computation of $M_{\psi,X}$ and f
$\Rightarrow$ f can be as small as $\Lambda/10$ $\rightarrow$ large hierarchy could explain that no new states have been observed so far at LHC

- Computation of the composite meson masses (consistent with lattice results)
$\Rightarrow$ spectrum belong to multi-TeV range but few states can be relatively light
  - EW and coloured pNGBs including η_0
  - η' for small κ_B/κ_A
  - σ for small ξ

- Only few parameters (ξ, κ_{A6}/κ_A, κ_B/κ_A, N, m_X) if current-current hypothesis is assumed $\Rightarrow$ Phenomenologically simple

Main limitation: absence of interactions with SM fermion fields

- Baryon masses could be used as input parameters for more effective approaches where mixing with light top partners is explicitly included
$\Rightarrow$ Exotic decays of VLQs could significantly affect experimental bounds
  - $T \to \eta_0 t$ with large branching ratio
  - $\widetilde{T}_5 \to \eta t$ with $Br = 1$ [1712.XXXXX, Bizot, Caciapaglia, Flacke]
  - $X_{5/3} \to \pi_6^c t, \cdots$

- Consider other UV completions
$\Rightarrow$ $f \sim N$ imply lighter composite resonances

- Apply NJL to minimal fundamental partial compositeness
$\Rightarrow$ $B = (S\psi)$: easy to compute top partners masses [Sanino, Strumia, Tesi, '16]

- Other applications of NJL techniques to composite (Higgs) models?

Thanks for your attention!

Four-fermions operators couplings may be related
⇒ Prediction of relative strength between the various physical channels (works well in QCD)

- Start from $Sp(2N)$ current-current operators: encode UV dynamics in 'ladder' approximation, that holds when N is (moderately) large
- Use Fierz transformations to generate various operators

$$\mathcal{L}_{UV} = g_{HC}\mathcal{J}_{\psi}^{\mu I}\mathcal{G}_{\mu I} \qquad \mathcal{J}_{\psi}^{\mu I} = \psi\left(\Omega T^{I}\right)\sigma^{\mu}\overline{\psi}$$

Assume that confining strong dynamics can be described (1^{rst} approximation) by exchange of one hypergluon which acquired a dynamical mass
⇒ 'Ladder' approximation strong dynamics generates $Sp(2N)$ current-current operators

$$\mathcal{L}_{eff} = \frac{\kappa_{UV}}{2N}\mathcal{J}_{\psi}^{\mu I}\mathcal{J}_{\psi\mu}^{I} \qquad \kappa_{UV}/(2N) \sim g_{HC}^{2}/\Lambda^{2} \quad (g_{HC} \sim 1/\sqrt{2N})$$

Lorentz and SU(N) for the fundamental (flavour) Fierz transformations are very well-known but not $Sp(2N)$ that we derived

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$Sp(2N)$ Fierz matrix for the fundamental representation:

$$\begin{pmatrix} (\Omega T^0)_{ij}(\Omega T^0)_{kl} \\ \sum_I (\Omega T^I)_{ij}(\Omega T^I)_{kl} \\ \sum_{\hat{I}} (\Omega T^{\hat{I}})_{ij}(\Omega T^{\hat{I}})_{kl} \end{pmatrix} = \begin{pmatrix} \frac{1}{2N} & \frac{1}{2N} & \frac{1}{2N} \\ \frac{2N+1}{2} & -\frac{1}{2} & \frac{1}{2} \\ \frac{(2N+1)(N-1)}{2N} & \frac{N-1}{2N} & -\frac{N+1}{2N} \end{pmatrix} \begin{pmatrix} (\Omega T^0)_{il}(\Omega T^0)_{kj} \\ \sum_I (\Omega T^I)_{il}(\Omega T^I)_{kj} \\ \sum_{\hat{I}} (\Omega T^{\hat{I}})_{il}(\Omega T^{\hat{I}})_{kj} \end{pmatrix},$$

The fate of the $SU(4)$ symmetry

- The model is a vector-like gauge theory: all fermions ψ can be made massive ($m_\psi \psi\psi$), while preserving the gauge hypercolour symmetry $G_c = Sp(2N)$

Three cases in vector-like theories: [Peskin, '80]

- $G = SU(N_f)_L \times SU(N_f)_R$ and $H_m = SU(N_f)_V$ (complex rep. of $\mathcal{G}$)
- $G = SU(2N_f)$ and $H_m = SO(2N_f)$ (real rep.)
  $H_m = Sp(2N_f)$ (pseudo-real rep.)

- Vafa-Witten theorem: The flavour subgroup H of G preserved by m_ψ can not be spontaneously broken $\Rightarrow$ If $SU(4)$ broken, it is broken down to $Sp(4)$

- 't Hooft anomaly matching:

Any global UV anomaly (generated by the hyperfermions ψ) must be matched in the IR, either by massless spin-1/2 baryons or Goldstone boson

ψ's can not form baryons because they are in pseudo-real hypercolour irreps
$\Rightarrow$ $SU(4)$ **unavoidably** spontaneously broken

$$d^{AB\hat{C}} = 2Tr[\{T^A, T^B\}T^{\hat{C}}]$$

$SU(4)$ broken ($T^{\hat{A}}$) and unbroken (T^A) generators combine in non-zero anomaly coefficients $\Rightarrow$ Global anomalies

$$(\psi^a\psi^b) \equiv \psi_i^a \Omega_{ij} \psi_j^b$$

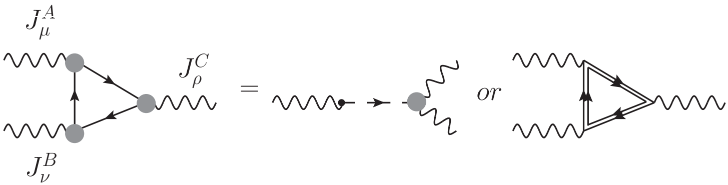
The unique invariant tensor of $Sp(2N)$ is two-index antisymmetric
$\Rightarrow$ $SU(4)$-flavour contraction also antisymmetric
($4 \times 4 = 6_A + 10_S$)

Trilinear baryons: $\Psi^{abf} = (\psi^a \psi^b X^f)$, $\Psi^{ab}_f = (\psi^a \psi^b \overline{X}_f)$ $\Psi^{af}_b = (\psi^a \overline{\psi}_b X^f)$
$\Psi^{fgh} = (X^f X^g X^h)$, $\Psi^{fg}_h = (X^f X^g \overline{X}_h)$

Anomaly matching condition:

$$\sum_{i=\psi,X} n_i A(r_i) = \sum_{i=baryon} n_{i'} A(r_i), \qquad 2\,Tr[T_r^{\hat{A}}\{T_r^B, T_r^C\}] = A(r) d^{\hat{A}BC}$$

- $SU(4)^3$: Matching impossible for $N \neq 8n \Rightarrow SU(4)$ breaks to $Sp(4)$ and one expects non-zero condensate $\langle\psi\psi\rangle \neq 0$
- $SU(6)^3$: Matching always possible $\Rightarrow SU(6)$ may not break to $SO(6)$ and the condensate $\langle XX \rangle$ may vanish or not
- $SU(4)^2 \times U(1), SU(6)^2 \times U(1), U(1)^3$: $U(1)$ most likely broken by $\langle\psi\psi\rangle$



Radiative contributions to the coloured pNGBs

Gauging explictly breaks G and induce radiative mass to NGBs:

$$\Delta M^2_{G_{\hat{A}}} = -\frac{3}{4\pi}\frac{1}{F^2_G}\frac{g^2_{\mathcal{W}}}{4\pi}\times\int_0^\infty dQ^2\, Q^2\, \Pi_{V\text{-}A}(-Q^2)\times\left[\sum_{\hat{B}}\left(f^{\hat{A}W\hat{B}}\right)^2 - \sum_{B}\left(f^{\hat{A}\hat{W}B}\right)^2\right]$$

$f^{abc} = 2iTr(T^a[T^b, T^c])$

$T^{\mathcal{W}} = T^W + T^{\hat{W}}$ gauged generators, $T^{W,\hat{W}}$ linear combination of $T^{A,\hat{A}}$

As $G_{SM} \subset H \to f^{\hat{A}\hat{W}B} = 0 (T^{\hat{W}} = 0) \Rightarrow$ Always positive contributionin CHMs that can not break EW symmetry

Coloured pNGBs masses

Coloured pNGBs receive mass from gluon loops:

$$\underline{\text{Octet}} : \Delta M^2_{O_c} = -\frac{3}{4\pi}\frac{1}{F^2_{G_c}}\int_0^\infty dQ^2\, Q^2\, \Pi^X_{V\text{-}A}(-Q^2)\times\frac{3}{4\pi}g_s^2$$

$$\underline{\text{Sextet}} : \Delta M^2_{S_c} = -\frac{3}{4\pi}\frac{1}{F^2_{G_c}}\int_0^\infty dQ^2\, Q^2\, \Pi^X_{V\text{-}A}(-Q^2)\times\frac{1}{4\pi}\left(\frac{10}{3}g_s^2 + \frac{16}{9}g'^2\right)$$

$\Rightarrow$ Enough to comply with direct searches even for $f = 1$ TeV
(and even for $m_X = 0$ contarry to the common expectation)