Bistability in the sine-Gordon chain

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This is a sketch of the results obtained on the sine-Gordon model (continuous and discrete) and published in :
Bistability in the sine-Gordon equation: the ideal switch
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If $u_n(t)$ represents the deviation (angle) of the n^{th} pendulum, the equation for the chain reads

$$\ddot{u}_n + \gamma \dot{u}_n - c^2 \left(u_{n+1} + u_{n-1} - 2u_n\right) + \omega_0^2 \sin u_n = 0, \quad n = 1 \cdots N, \tag{1}$$

where harmonic (and homogeneous) interaction is assumed between neighbors. The parameter ω_0 is actually the eigenfrequency of an isolated pendulum and γ is a phenomenological damping coefficient. Dot means differentiation with respect to time and c^2 is the coupling torque constant.

The boundary-value problem associated to that equation is a forced extremity

$$u_0(t) = a \ \cos(\Omega t)$$

and a free end

$$u_{N+1}(t) = u_N(t).$$

Bistability is simply the property that for a given input amplitude a, there may exist two (or more) different output amplitudes

$$A = \max_t \{u_N(t)\}$$

of a permanent stationary regime.

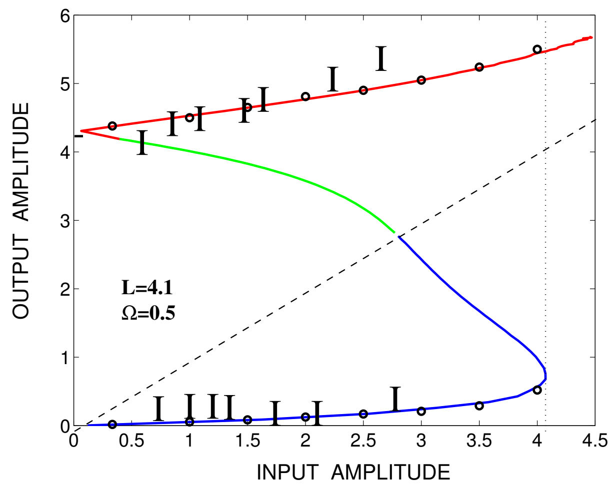

Figure 1: Plot of the curves output amplitude A as a function of the input a for $\Omega = 0.6$ (in reduced units). This plot shows the hysteris loop obtained by the explicit solutions of the continuous limit boundary value problem (full lines) compared to numerical simulations of the discrete equation (dots) and to experimental measurements with the chain of the movie (bars).

For instance the displayed movie shows such two regimes obtained for the same forcing amplitude. The bifurcation in that case is obtained by an external perturbation (a kick with the magic stick).